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P A M

New Developments in Performance Assessment and Benchmarking

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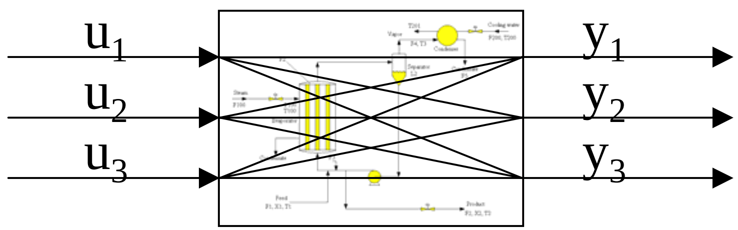
Presentation Outline

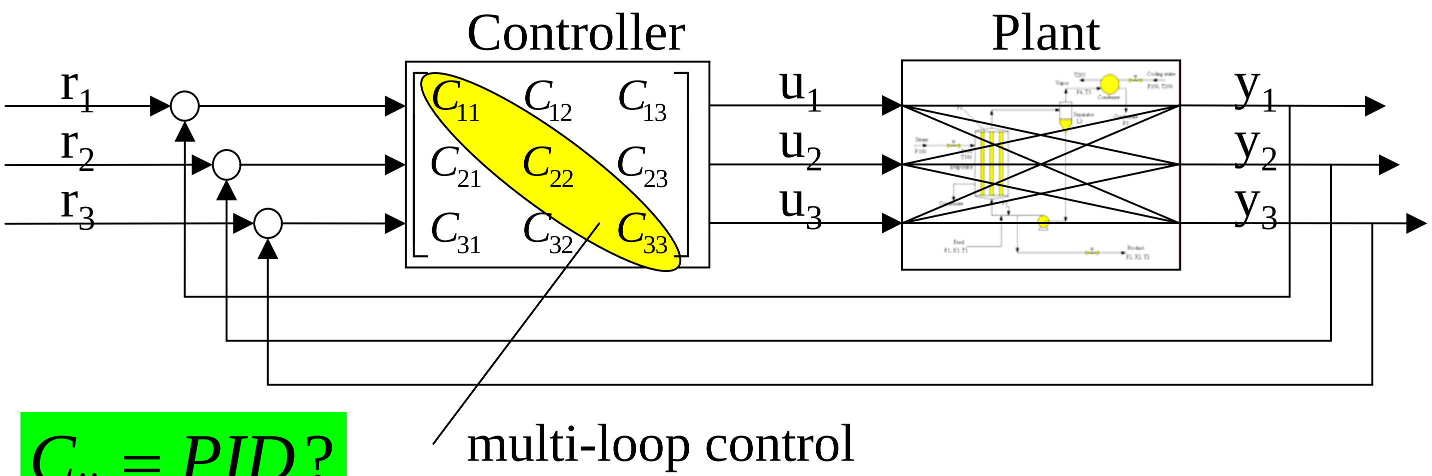
- Introduction
- Multivariable controller design and benchmarking
- Restricted-structure benchmarking in MIMO case
- Structure assessment
- Simulation example
- Summary

Introduction: MIMO Benchmarking

- Minimum variance benchmark and “Harris index” have become widespread over the last decade
- MV control rarely used but MV benchmark still useful
- GMV controller proposed as an alternative benchmark
- So far focus mostly on SISO performance
- But most processes involve interactions between a number of control loops ?   MIMO benchmark better



Controller Structures



$C_{ij} = PID?$

- Benchmarking against optimal controllers does not take into account the existing controller structure
- Restricted-structure controller approach: compare against the best actually achievable controller
- MIMO RS approach may have other useful applications such as optimal I/O pairing and structure assessment

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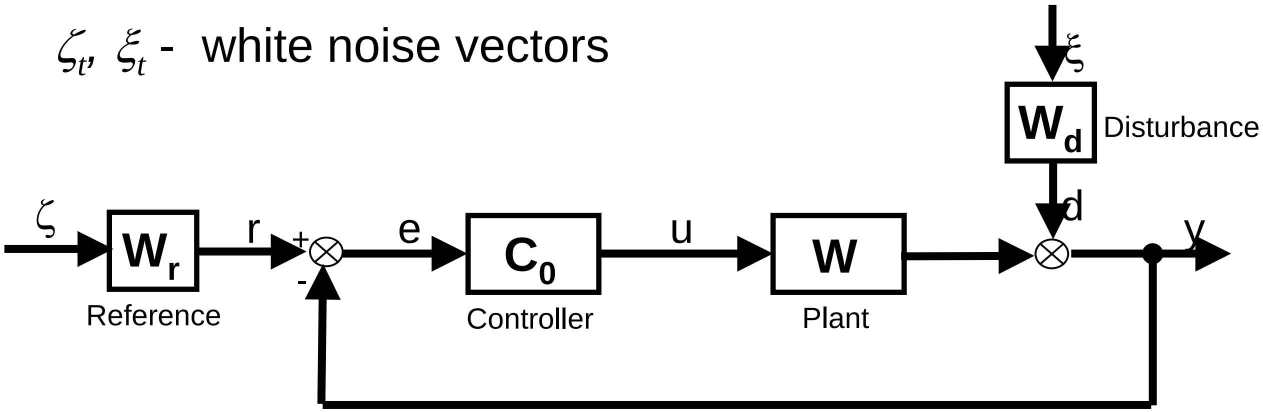
Multivariable Control System Description

ζ_t, ξ_t - white noise vectors



System matrix fractions

$$W(z^{-1}) = A^{-1}(z^{-1})B(z^{-1})$$

$$W_r(z^{-1}) = A^{-1}(z^{-1})E_r(z^{-1})$$

$$W_d(z^{-1}) = A^{-1}(z^{-1})C_d(z^{-1})$$

$$e_t = r_t - y_t = S_r(z^{-1})(r_t - d_t)$$ - control error

$$u_t = C_0(z^{-1})S_r(z^{-1})(r_t - d_t)$$ - control signal

$$S_r = (I_r + WC_0)^{-1}$$

sensitivity function

Plant Delay Structure

Interactor matrix (IM) – generalization of the scalar time delay

Fundamental performance limitation in the system

SISO	MIMO
$y(t) = T(z^{-1})u(t) = z^{-k}\tilde{T}(z^{-1})u(t)$ $\lim\limits_{z^{-1}\to 0} z^k \cdot T(z^{-1}) = k, \quad \mathrm{k} \neq 0$	$Y(t) = T(z^{-1})Y(t) = D^{-1}\tilde{T}(z^{-1})U(t)$ $\lim\limits_{z^{-1}\to 0} D(z)\cdot T(z^{-1}) = K, \quad \text{K finite and full rank}$ $\det\{D(z)\} = z^k, \quad \text{k - number of infinite zeros}$

- Interactor matrix characterizes the infinite zeros of the system
- It is often diagonal, but generally it may be a full matrix
- Interactor matrix of the system is not unique:
  - **lower triangular** IM (Wolovich and Falb 1976)
  - **nilpotent** IM (Rogozinski et al. 1987)
  - **unitary** IM (Peng and Kinnaert 1992)

Performance Criteria

GMV cost function: $J_{GMV} = Var\{\phi_t\}, \quad \phi_t = P_c e_t + F_c u_t$

LQG cost function: $J_{LQG} = \frac{1}{2\pi j} \oint_{|z|=1} trace\{Q_c \Phi_{ee} + R_c \Phi_{uu}\} \frac{dz}{z}$

	GMV	LQG
Design	+ simpler - restriction on the weightings	+ guaranteed stability - more involved
Benchmarking	+ data driven - interactor matrix needs to be known or estimated	+ “classical” optimal benchmark - full model needed

GMV Control: Generalized Plant

Minimum Variance Control:

$$y_t = D^{-1}\tilde{T}u_t + N\xi_t \qquad \Rightarrow \qquad \tilde{\phi}_t = q^{-k}Dy_t = q^{-k}\tilde{T}u_t + \underbrace{q^{-k}DN}_{\tilde{N}}\xi_t$$

Generalized Minimum Variance Control:

$$\phi_t = P_c e_t + F_c u_t = P_c(D^{-1}\tilde{T}u_t + N\xi_t) + F_c u_t = (P_c D^{-1}\tilde{T} + F_c)u_t + P_c N\xi_t =$$

$$= D_g^{-1}\tilde{T}_g u_t + N_g \xi_t$$

D_g unitary

Interactor filtering: $\tilde{\phi}_t = q^{-k}D_g\phi_t = q^{-k}\tilde{T}_g u_t + \tilde{N}_g\xi_t$

$$Var\{\phi_t\} = Var\{\tilde{\phi}_t\}$$

Multivariable GMV Controller

Main result:

$$J = J_{\min} + J_0$$

$$J_{\min} = \frac{1}{2\pi j} \oint_{|z|=1} trace\{F^* F\} \frac{dz}{z}$$

minimum achievable value

suboptimal term

$$J_0 = \frac{1}{2\pi j} \oint_{|z|=1} trace\{X^* X\} \frac{dz}{z}$$

$$D_f^{-1} A P_{cd} = A_2 D_2^{-1} \qquad F A_2 + z^{-k} G = P_{cn} D_2$$

$$D_f^{-1} B F_{cd} = B_2 D_3^{-1} \qquad F B_2 - z^{-k} H = F_{cn} D_3$$

Diophantine equations

$$X = (G D_2^{-1} P_{cd}^{-1} C_{0d} - H D_3^{-1} F_{cd}^{-1} C_{0n})(A C_{0d} + B C_{0n})^{-1} D_f$$

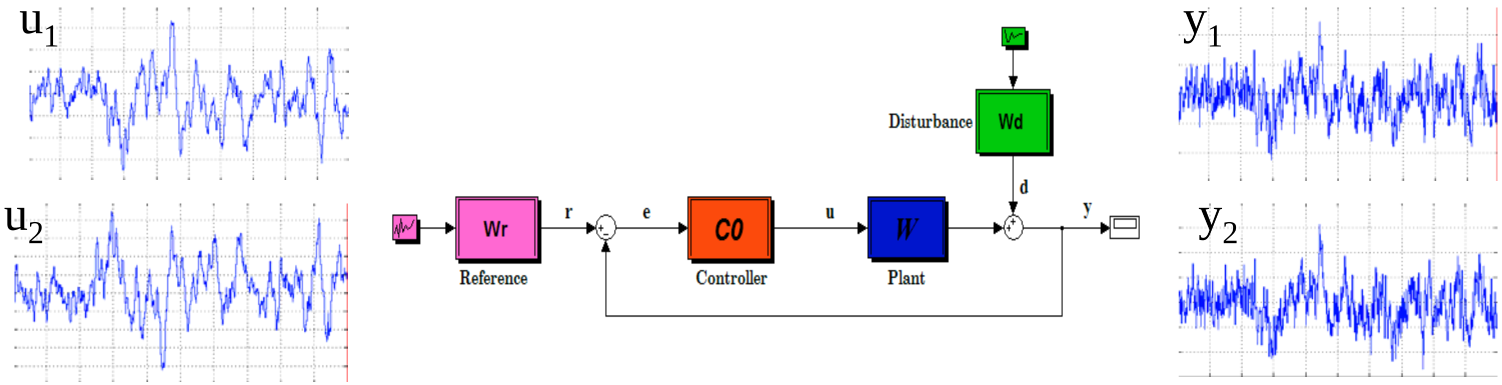
$$C_0 = F_{cd} D_3 H^{-1} G D_2^{-1} P_{cd}^{-1}$$

optimal controller

D_f: filter spectral factor

MIMO GMV Benchmarking

Benchmarking procedure – FCOR algorithm (Huang and Shah 1999)



$$\phi_t = P_c e_t + F_c u_t$$

filtering

$$\epsilon_t = A(z^{-1})\phi_t \quad \Leftarrow \quad \phi_t = A^{-1}(z^{-1})\epsilon_t$$

whitening

$$E[\phi_{t-j}\epsilon_t] = F_j, \quad j = 0,\ldots,k-1$$

correlation

$$J_{\min}$$

$$\eta = \frac{J_{\min}}{Var[\phi_t]}$$

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Motivation

- Often a need for relatively unskilled staff to tune controllers and this implies low order controllers needed.
- Low order controllers often have improved robustness properties relative to high order designs.
- Often a requirement to have the good control action of advanced designs within a controller structure that is simple.
- Nonlinear systems can be linearised at operating points and multiple model RS controllers designed
- Optimal RS controllers can be used as realistic benchmarks; however, model needed in this case

Reduced Controller Structures

SISO level	MIMO level
Reduced order: $C_0(s) = \dfrac{c_{n0} + c_{n1}s + \ldots + c_{np}s^p}{c_{d0} + c_{d1}s + \ldots + c_{dv}s^v}$ where $v = p$ is less than the order of the system (plus weightings)	***Diagonal:*** $\begin{bmatrix} C_0 & & \\ & C_1 & \\ & & C_2 \end{bmatrix}$
Lead lag: $C_0(s) = \dfrac{(c_{n0} + c_{n1}s)(c_{n2} + c_{n3}s)}{(c_{d0} + c_{d1}s)(c_{d2} + c_{d3}s)}$	***Triangular:*** $\begin{bmatrix} C_0 & C_{01} & C_{02} \\ & C_1 & C_{12} \\ & & C_2 \end{bmatrix}$
PID: $C_0(s) = k_0 + k_1 / s + k_2 s$	***Full:*** $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$
Filtered PID: $C_0(s) = k_0 + \dfrac{k_1}{s} + \dfrac{k_2 s}{\tau s + 1}$	

The assumption must be made that a stabilising control law exists for the assumed controller structure.

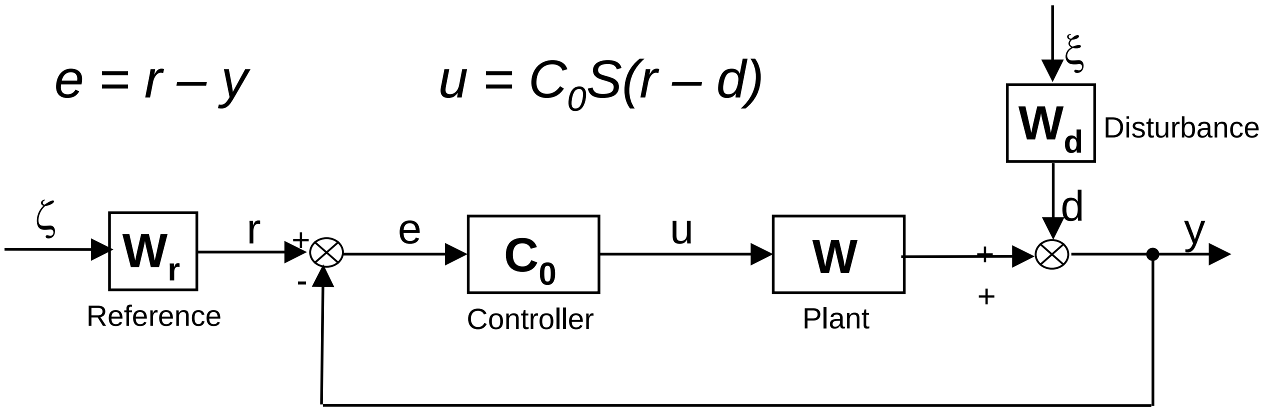
Restricted-Structure Design Issues

- MIMO structure: choice of input/output variables, I/O pairing, additional couplings (can use the existing structure)
- Restricted controller structure for each element of the transfer-function matrix
- Optimality criterion - e.g. GMV, LQG, H cost functions
- Optimization algorithm (how to find the “optimal” controller parameters)

RS-LQG Control – SISO Case

Stochastic System Description

$e = r - y$ $\qquad$ $u = C_0 S(r - d)$



$$W(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$$

$$W_d(z^{-1}) = \frac{C_d(z^{-1})}{A(z^{-1})}$$

$$W_r(z^{-1}) = \frac{E_r(z^{-1})}{A(z^{-1})}$$

$$S = \frac{1}{1 + WC_0}$$

sensitivity function

ξ, ζ - independent white noise sequences

LQG cost function to minimize:

$$J = \frac{1}{2\pi j} \oint_{|z|=1} \{Q_c \Phi_{ee} + R_c \Phi_{uu}\} \frac{dz}{z}$$

Dynamic weightings

$$Q_c = H_q^* H_q = \frac{B_q^* B_q}{A_q^* A_q}$$

$$R_c = H_r^* H_r = \frac{B_r^* B_r}{A_r^* A_r}$$

RS-LQG Control – SISO Case

Step 1: Design full-order optimal controller

- Introduce an innovations model for the signal: $f = r - d = Y_f \varepsilon = \dfrac{D_f}{A}\varepsilon$

zero mean white noise of unit variance

$$D_f D_f^* = E_r E_r^* + C_d C_d^*$$ - noise spectral factor

⇒ $e = SY_f\varepsilon$ and $u = C_0 SY_f\varepsilon$

$$D_c D_c^* = B^* A_r^* B_q^* B_q A_r B + A^* A_q^* B_r^* B_r A_q A$$

Spectral factorization

$$\overline{D}_c G + FAA_q = \overline{BA_r} Q_n D_f$$

$$\overline{D}_c H - FBA_r = \overline{AA_q} R_n D_f$$

Diophantine equations

$$J = J_{\min} + J_0$$

Suboptimal component:

$$J_0 = \frac{1}{2\pi j}\oint_{|z|=1} T_0 T_0^* \frac{dz}{z}$$

$$T_0 = \frac{HA_q C_{0n} - GA_r C_{0d}}{A_q A_r (AC_{0d} + BC_{0n})}$$

RS-LQG Control – SISO Case

Step 2: Restricted-structure optimization problem

The optimal controller must be chosen in such a way that J_0 is minimized:

$$\min_{\{k_0, k_1, k_2\}} J_0 \qquad T_0 = \frac{HA_q C_{0n} - GA_r C_{0d}}{A_q A_r (AC_{0d} + BC_{0n})}$$

subject to the controller structure: $C_0(z^{-1}) = k_0 + \dfrac{k_1}{1 - z^{-1}} + \dfrac{k_2(1 - z^{-1})}{1 - \alpha z^{-1}}$

$$J_0 = \frac{1}{2\pi j} \oint_{|z|=1} T_0(z^{-1}) T_0^*(z) z^{-1} dz = \frac{T_s}{2\pi} \int_0^{2\pi/T_s} T_0(e^{j\omega T_s}) T_0(e^{-j\omega T_s}) d\omega$$

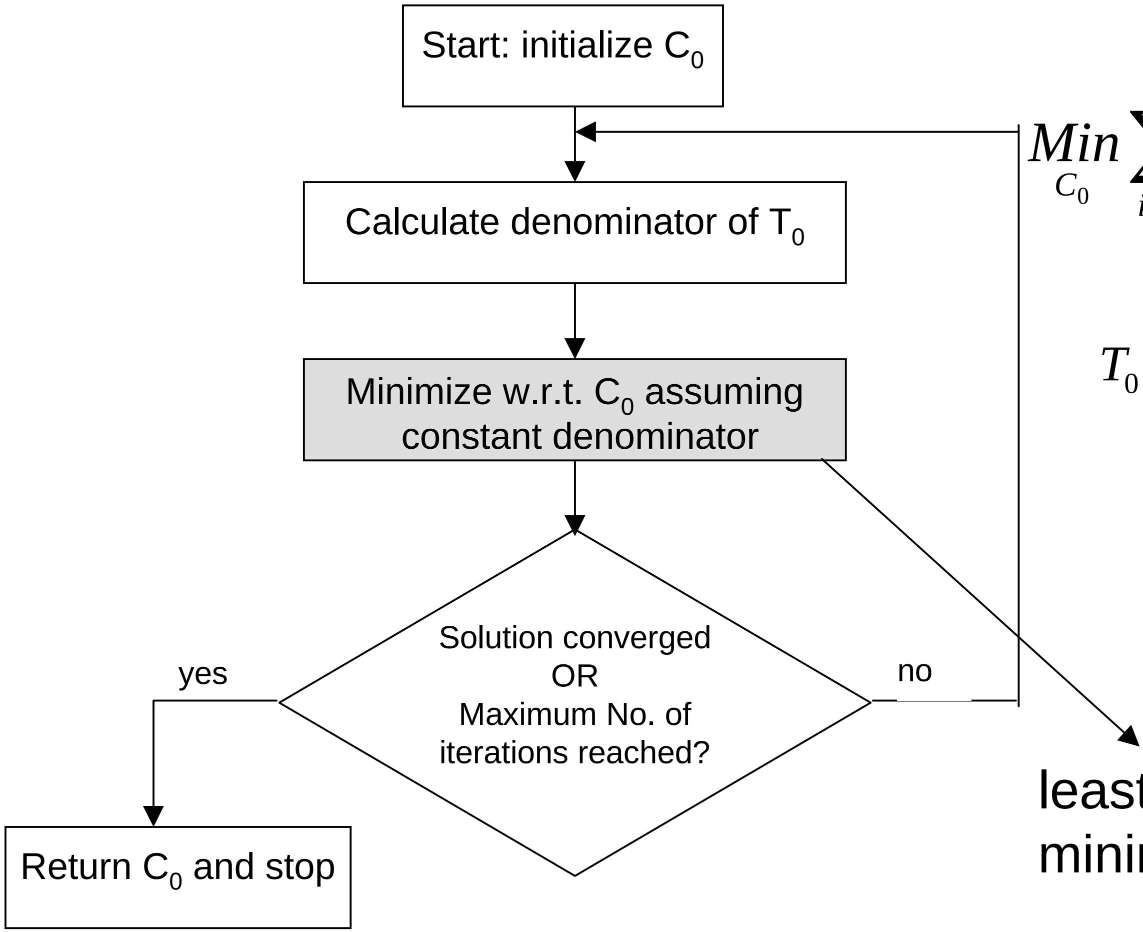
$$\approx \frac{T_s}{2\pi} \sum_{i=1}^{N} T_0(e^{j\omega_i T_s}) T_0(e^{-j\omega_i T_s}) \cdot \Delta\omega_i$$

Optimization

$$\underset{\{k_0, k_1, k_2\}}{Min} \; J_0$$

- This is a nonlinear optimization problem and generally does not have a closed analytical solution
- Iterative numerical solution required
  - Successive approximation / Edmunds' algorithm
  - Gradient methods / Optimization toolbox
  - Search algorithms (genetic, evolutionary etc.)

RS-LQG: Successive approximation



$$\underset{C_0}{Min} \sum_{i=1}^{N} T_0(e^{j\omega_i T_s}) T_0(e^{-j\omega_i T_s}) \Delta\omega_i$$

$$T_0 = \frac{HA_q C_{0n} - GA_r C_{0d}}{A_q A_r (AC_{0d} + BC_{0n})}$$

least squares minimization

RS- LQG: Successive Approximation

Minimize w.r.t. C_0 assuming constant denominator

$$\underset{C_0}{Min}\sum_{i=1}^{N} T_0(e^{j\omega_i T_s})T_0(e^{-j\omega_i T_s})\Delta\omega_i \qquad T_0 = \frac{HA_qC_{0n} - GA_rC_{0d}}{A_qA_r(AC_{0d} + BC_{0n})}$$

PID Controller (linear w.r.t. parameters):

$$C_{0n}(z^{-1}) = \alpha_0(z^{-1})k_0 + \alpha_1(z^{-1})k_1 + \alpha_2(z^{-1})k_2 \qquad C_{0d} = (1 - z^{-1})(1 - \alpha z^{-1})$$

- Choose the frequency range and number of frequency points N
- For all frequency points calculate all elements of **F** and **L**: $T_0 = Fx - L$

where:

$$F = \begin{bmatrix} f_{11}^r(\omega_1) & f_{12}^r(\omega_1) & f_{13}^r(\omega_1) \\ \vdots & \vdots & \vdots \\ f_{11}^r(\omega_N) & f_{12}^r(\omega_N) & f_{13}^r(\omega_N) \\ f_{11}^i(\omega_1) & f_{12}^i(\omega_1) & f_{13}^i(\omega_1) \\ \vdots & \vdots & \vdots \\ f_{11}^i(\omega_N) & f_{12}^i(\omega_N) & f_{13}^i(\omega_N) \end{bmatrix} \qquad L = \begin{bmatrix} L_{11}^r(\omega_1) \\ \vdots \\ L_{11}^r(\omega_N) \\ L_{11}^i(\omega_1) \\ \vdots \\ L_{11}^i(\omega_N) \end{bmatrix} \qquad x = [k_0\ k_1\ k_2]^T$$

- Calculate optimal **x**:

$$\boxed{x^{opt} = (F^TF)^{-1}F^TL}$$

Multivariable RS-LQG Control Design

• Similar procedure can be applied to design the optimal RS-LQG controller in MIMO case

• The suboptimal cost function component to be minimized becomes:

$$J_0 = \frac{1}{2\pi j} \oint_{|z|=1} trace\{X^* X\} \frac{dz}{z} \qquad C_0(z^{-1}) = K_0 + \frac{K_1}{1-z^{-1}} + \frac{K_2(1-z^{-1})}{1-\alpha z^{-1}}$$

where

$$X = (GD_2^{-1}A_q^{-1}C_{0d} - HD_3^{-1}A_r^{-1}C_{0n})(AC_{0d} + BC_{0n})^{-1}D_f$$

can be represented as a transfer-function matrix.

• J_0 is a scalar so gradient and search algorithms can still be applied to find the optimal solution

• the successive approximation algorithm can also be generalized and least squares solution used

Multivariable RS-LQG Benchmarking

The procedure:

(1) Specify the restricted structure – e.g. multi-loop with PID controllers

(2) Find the RS controller by minimizing J_0

(3) Calculate the minimum value of J_0

$$J_0^{opt} = \frac{1}{2\pi j} \oint_{|z|=1} trace\{X_{opt}^* X_{opt}\} \frac{dz}{z}$$

(4) Calculate this value for the existing controller structure

$$J_0^{act} = \frac{1}{2\pi j} \oint_{|z|=1} trace\{X_{act}^* X_{act}\} \frac{dz}{z}$$

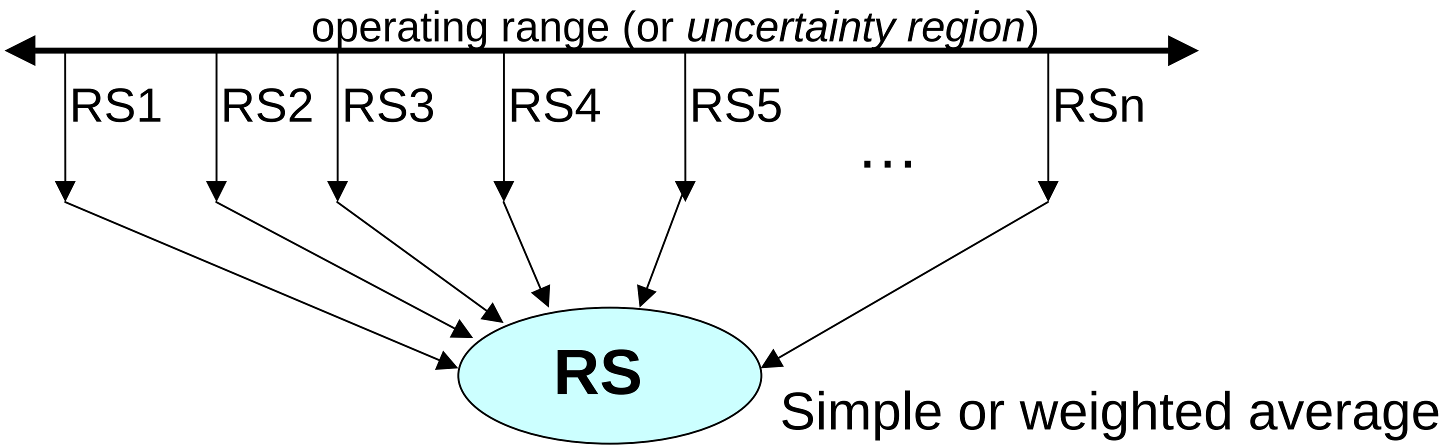
(5) Calculate Controller Performance Index

$$\kappa = \frac{J_{\min} + J_0^{opt}}{J_{\min} + J_0^{act}}$$

Multiple models

- Above algorithms valid for linear system description
- However, real systems are inherently nonlinear
- One possible solution:

> use a number of linear models defined for different operating points



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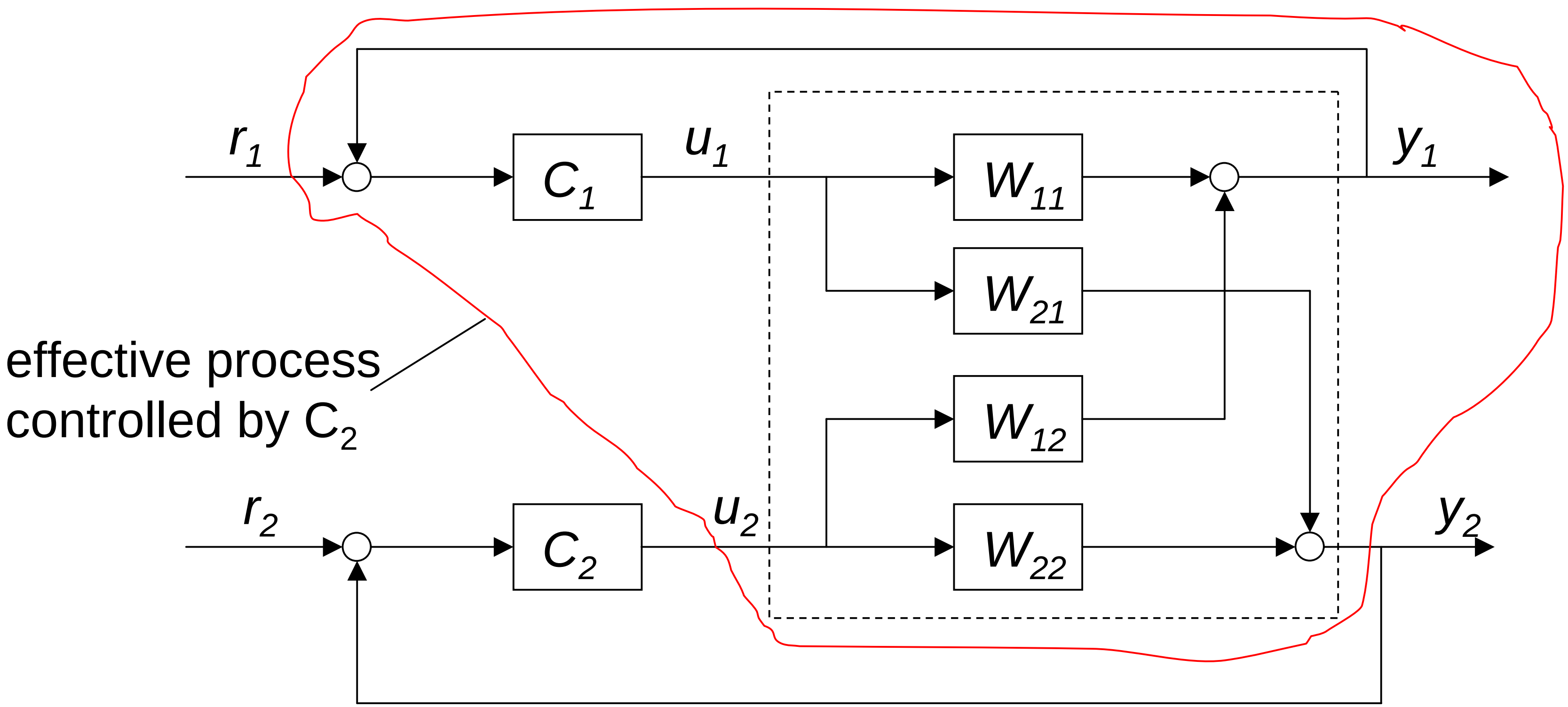
Multivariable Interactive Control System



Issues in the Multivariable Control Design

- Selection of the variables to be controlled
- Choice of the manipulated variables
- Interactions between loops
- Self-regulatory v. non-self regulatory loops
- I/O pairing
- Joint stability regions may shrink!
- Decoupling v. multivariable control

I/O Pairing and Relative Gain Array

• Problem of the "best" choice of input/output pairs for a multi-loop control system

• Relative Gain Array (Bristol 1966) widely used in industry

• Steady-state gains required (deviations about operating point) – obtained from step tests or from the linear model

• Relative Gain Array defined as

$$RGA = (K^{-1})^T \otimes K$$

$\otimes$ → element-by-element product

where *K* is the matrix of static gains between all inputs and all outputs

I/O Pairing and Relative Gain Array

- RGA close to unity = desired pairing with no interactions
- RGA close to zero = pairing should be avoided
- All elements approx. equal = strongest possible interactions
- Results might be against "conventional wisdom"
- Takes into account only steady-state information

Application of RS Design to I/O Pairing

For a 3x3 MIMO system, the following multi-loop PID control configurations are possible:

$$A)\begin{bmatrix} PID & & \\ & PID & \\ & & PID \end{bmatrix} \quad B)\begin{bmatrix} PID & & \\ & & PID \\ & PID & \end{bmatrix} \quad C)\begin{bmatrix} & PID & \\ PID & & \\ & & PID \end{bmatrix}$$

$$D)\begin{bmatrix} & & PID \\ PID & & \\ & PID & \end{bmatrix} \quad E)\begin{bmatrix} & & PID \\ & PID & \\ PID & & \end{bmatrix} \quad F)\begin{bmatrix} & PID & \\ & & PID \\ PID & & \end{bmatrix}$$

- Using RS design technique it is possible to determine the optimal structure in terms of the specified cost function (takes into account also dynamic properties of the system)
- A means of optimally tuning the controllers
- Requirement: full dynamic model of the system

Application of RS Design to Structure Assessment

The controller structure is restricted on two levels, e.g.

$$a)\begin{bmatrix} PI & & \\ & PI & \\ & & PID \end{bmatrix} \qquad b)\begin{bmatrix} PI & PID & PD \\ & PI & P \\ & & PID \end{bmatrix} \qquad c)\begin{bmatrix} PID & PID & PID \\ PID & PID & PID \\ PID & PID & PID \end{bmatrix}$$

Multi-loop   Triangular   Full PID structure

- Possible to determine potential benefits that may result from adding additional controllers to the multi-loop system
- When already decided on a particular structure, the optimal controller parameters readily available

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System Model

<u>Example</u>: regulatory control of a simple 2x2 system with time delays (Huang & Shah 1999)

Plant model:

$$W = \begin{bmatrix} \dfrac{z^{-1}}{1-0.4z^{-1}} & \dfrac{K_{12}z^{-2}}{1-0.1z^{-1}} \\ \dfrac{0.3z^{-1}}{1-0.1z^{-1}} & \dfrac{z^{-2}}{1-0.8z^{-1}} \end{bmatrix}$$

Disturbance model:

$$W_d = \begin{bmatrix} \dfrac{1}{1-0.5z^{-1}} & \dfrac{-0.6}{1-0.5z^{-1}} \\ \dfrac{0.5}{1-0.5z^{-1}} & \dfrac{1}{1-0.5z^{-1}} \end{bmatrix}$$

Parameter K_{12} determines interaction between input 2 and output 1.

The plant has a general interactor matrix D:

$$D = \begin{bmatrix} -0.9578z & -0.2873z \\ 0.2873z^{2} & -0.9578z^{2} \end{bmatrix}$$

Estimation of the Minimum Variance

Existing controller:

$$C_0 = \begin{bmatrix} \frac{0.5-0.2z^{-1}}{1-0.5z^{-1}} & 0 \\ 0 & \frac{0.25-0.2z^{-1}}{(1-0.5z^{-1})(1+0.5z^{-1})} \end{bmatrix}$$

Multi-loop minimum variance controller (calculated for each loop separately)

Two choices of GMV weightings:

(1) MV weightings

$$P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(2) Static GMV weightings

$$P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_c = -D^{-1} \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$$

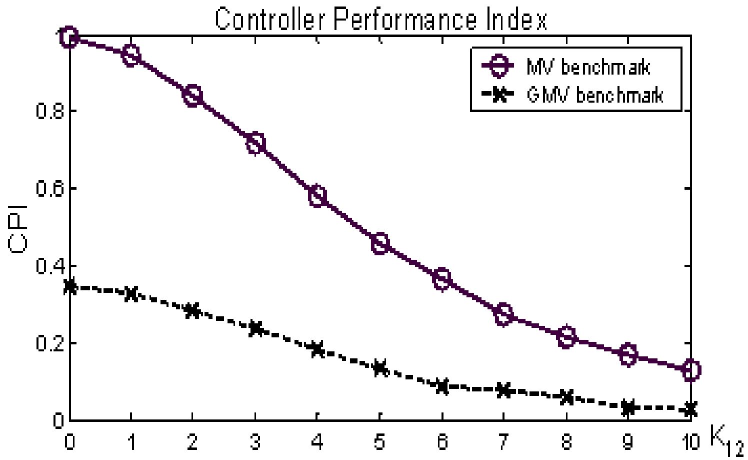
interactor

Benchmarking Results

Output and control variances for different values of K_{12}:

K_{12}	Controller	$Var[y_1]$	$Var[y_2]$	$Var[u_1]$	$Var[u_2]$
1	MV	1.3871	1.5514	0.7989	0.3020
	GMV	2.8245	2.1137	0.1822	0.0144
5	MV	1.3895	1.5514	8	8
	GMV	3.2303	1.9146	0.2078	0.0049
10	MV	1.4038	1.5514	8	8
	GMV	3.3870	1.8782	0.2409	0.0026

MV and GMV controller performance indices



- GMV criterion provides a means of balancing error and control variances
- It also makes the benchmark realizable (the plant is non-minimum phase for $K_{12} > 2$)
- MV benchmark close to 1 for small interactions
- the existing controller not so good if control variances considered

Restricted-Structure Controllers

Existing controller: filtered multi-loop PID tuned using Ziegler-Nichols rules separately for each loop.

The following RS-GMV controllers have been designed using the GMV weightings as in the previous slides:

RS full

$$\begin{bmatrix} PID & PID \\ PID & PID \end{bmatrix}$$

RS diagonal #1

$$\begin{bmatrix} PID & \\ & PID \end{bmatrix}$$

RS diagonal #2

$$\begin{bmatrix} & PID \\ PID & \end{bmatrix}$$

RS upper triangular

$$\begin{bmatrix} PID & PID \\ & PID \end{bmatrix}$$

RS lower triangular

$$\begin{bmatrix} PID & \\ PID & PID \end{bmatrix}$$

RS Benchmarking Results

The existing multi-loop PID has been assessed against the RS controllers:

Controller	J_0
Multi-loop PID	2.8026
RS full	0.0154
RS diagonal #1	0.0859
RS diagonal #2	0.5022
RS upper triangular	0.0357
RS lower triangular	0.0658

- The results show the potential for improving the current performance
- I/O pairing: OK
- If further reduction in variance required, additional feedback between output 1 and input 2 (rather than between output 2 and input 1) would bring greater improvement

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Summary

- Benchmarking multivariable loops
- Benchmarking multi-loop controllers
- Detecting underperforming loops
- Tuning guidelines for the existing controllers
- Determining the optimal controller structure (I/O pairing)