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Fifth Framework Programme

PAM

New Ideas in Performance Assessment and Benchmarking of Nonlinear Systems

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Presentation Outline

- Introduction
- Review of the linear GMV control theory
- Nonlinear GMV controller
- Performance assessment against NGMV controller
- Simulation example
- Summary

Introduction

- Minimum variance is the most popular stochastic benchmark: simple, meaningful, easy to calculate
- Comparison against the best possible <u>linear</u> controller
- However, all real plants are nonlinear
- Need for high performance control over wide operating range → nonlinear control
- Introduction of ***NGMV*** – a new simple nonlinear controller

Minimum Variance Control – a few dates

1970	*Åström*: MV controller for linear minimum phase plants
1971	*Clarke & Hastings-James*: Generalized MV criterion
1975	*Clarke & Gawthrop*: Self-tuning GMV controller
1988	*Grimble*: GMV control law revisited
1992	*Harris, Desborough*: MV benchmarking, "Harris" index
1999	*Huang & Shah*: MIMO MV benchmarking
2002	*Grimble*: GMV benchmarking
2003	***Grimble*: Nonlinear GMV control**

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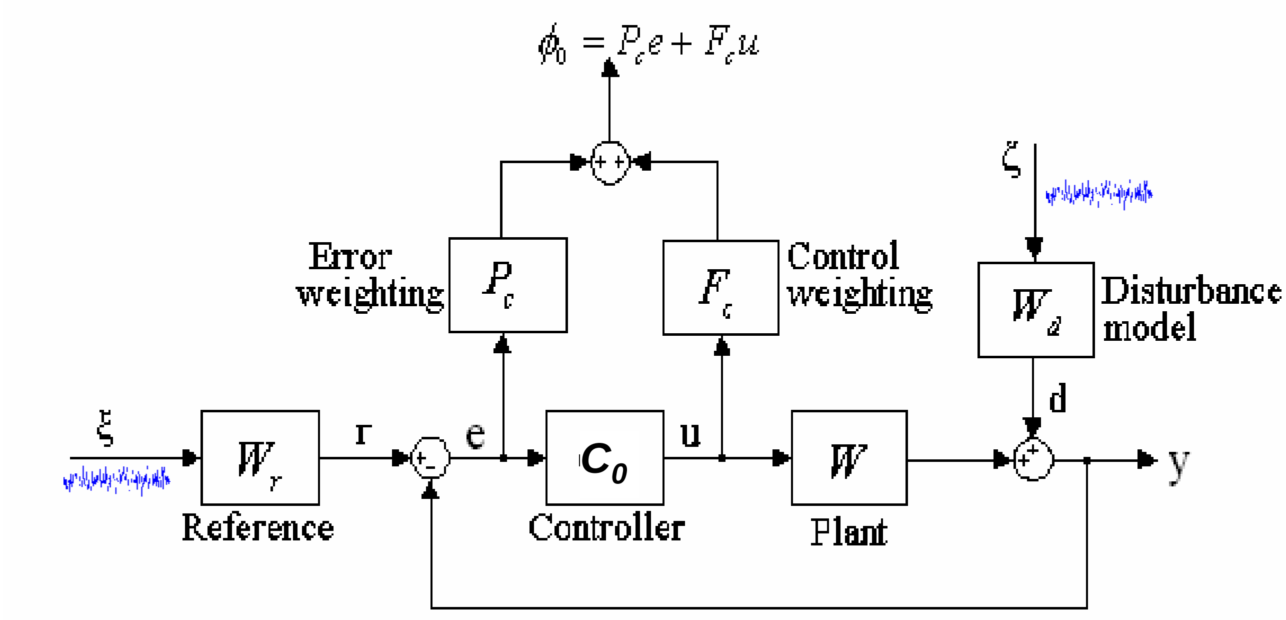
LTI System Model



$$W(z^{-1}) = z^{-k}\frac{B(z^{-1})}{A(z^{-1})}$$

$$W_r(z^{-1}) = \frac{E_r(z^{-1})}{A(z^{-1})}$$

$$W_d(z^{-1}) = \frac{C_d(z^{-1})}{A(z^{-1})}$$

$$S = (1 + WC_0)^{-1}$$

sensitivity function

$$e_t = r_t - y_t = S(z^{-1})(r_t - d_t)$$ - control error

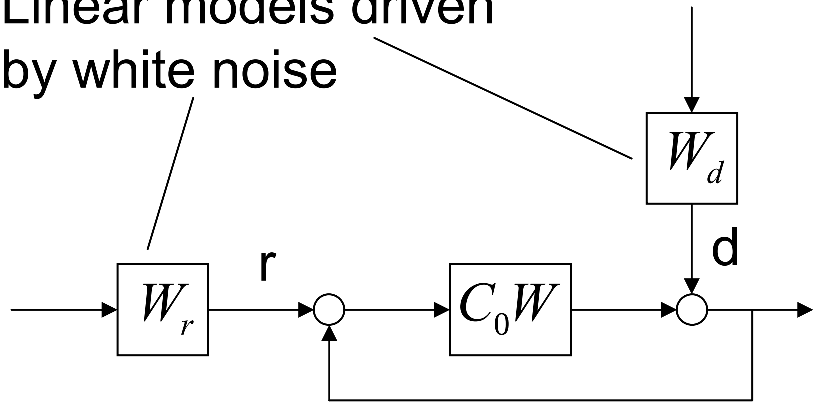
$$u_t = C_0(z^{-1})S(z^{-1})(r_t - d_t)$$ - control signal

ξ_t, ζ_t - Independent white noise sequences

Spectral Factorisation

Aim: combine all the stochastic inputs into one noise signal

Linear models driven by white noise



$$f(t) = Y_f(z^{-1})\varepsilon(t)$$

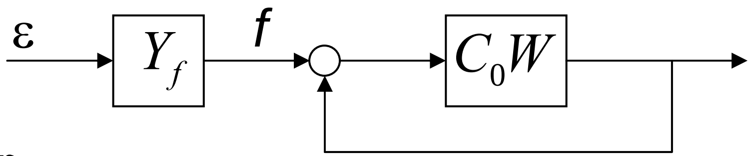
$\varepsilon(t)$ - zero mean white noise of unit variance

$$f(t) = r(t) - d(t)$$

$$\Phi_{ff} = \Phi_{rr} + \Phi_{dd} = W_r W_r^* + W_d W_d^*$$

$$Y_f Y_f^* = \Phi_{ff}$$

spectral factor



Minimum Variance Control

To minimize: output variance $J_{MV} = E[y^2(t)]$

$$y(t) = Wu(t-k) + Y_f\varepsilon(t)$$
$$= F\varepsilon(t) + Wu(t-k) + R\varepsilon(t-k)$$

statistically independent terms

$$Y_f = F + z^{-k}R$$

Diophantine equation

Optimal control: $u^{MV}(t) = -\frac{R}{W}\varepsilon(t) = -\frac{R}{WF}y(t)$

MV controller works when:

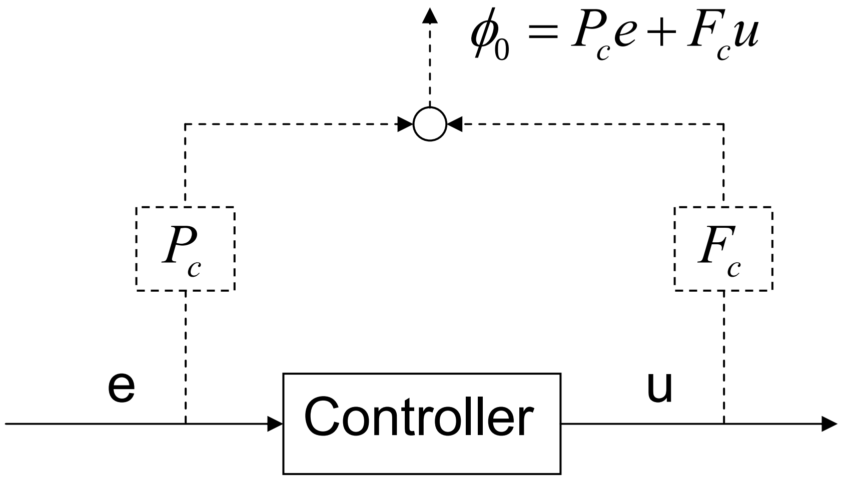
- the plant *W* is invertible (minimum-phase)
- reference and disturbance models are representative of the actual signals entering the system

Generalised Minimum Variance Criterion

To minimize: variance of the "generalized" output $\phi_0(t)$:

$$J_{GMV} = E[\phi_0^2(t)]$$

$$\phi_0(t) = P_c e(t) + F_c u(t)$$



Dynamic weightings

$$P_c = \frac{P_{cn}}{P_{cd}}, \quad F_c = z^{-k} \frac{F_{ck}}{F_{cd}}$$

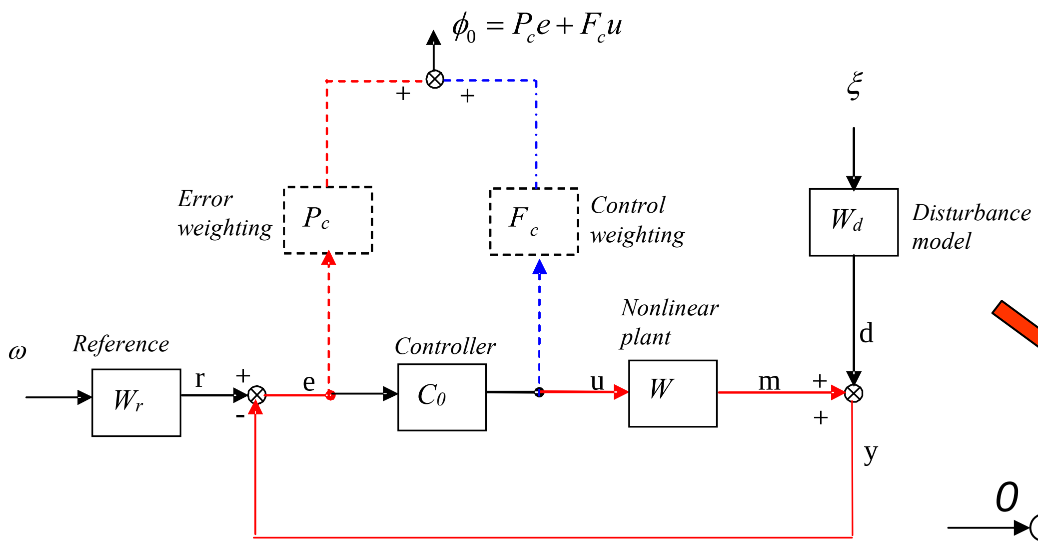
$e(t)$ affected by $u(t-k)$

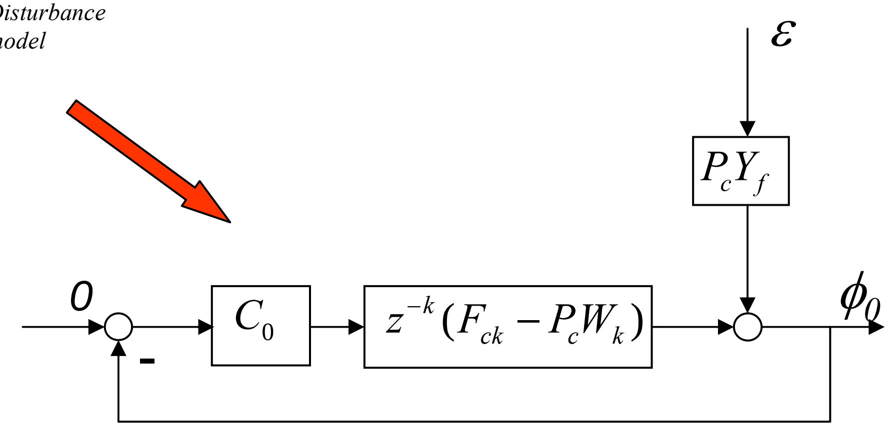
Generalized plant

GMV problem can be recast as an MV problem for the "generalized" plant:

$$\phi(t) = P_c(-z^{-k}W_k u(t) + Y_f \varepsilon(t)) + F_c u(t)$$

$$= z^{-k}(F_{ck} - P_c W_k)u(t) + P_c Y_f \varepsilon(t)$$





GMV Controller

MV control of the plant $(P_cW - F_c)$:

Diophantine equation

$$P_cY_f = F + z^{-k}R$$

$$\phi_0(t) = (P_cW - F_c)u(t) + P_cY_f\varepsilon(t)$$
$$= F\varepsilon(t) + (P_cW - F_c)u(t-k) + R\varepsilon(t-k)$$

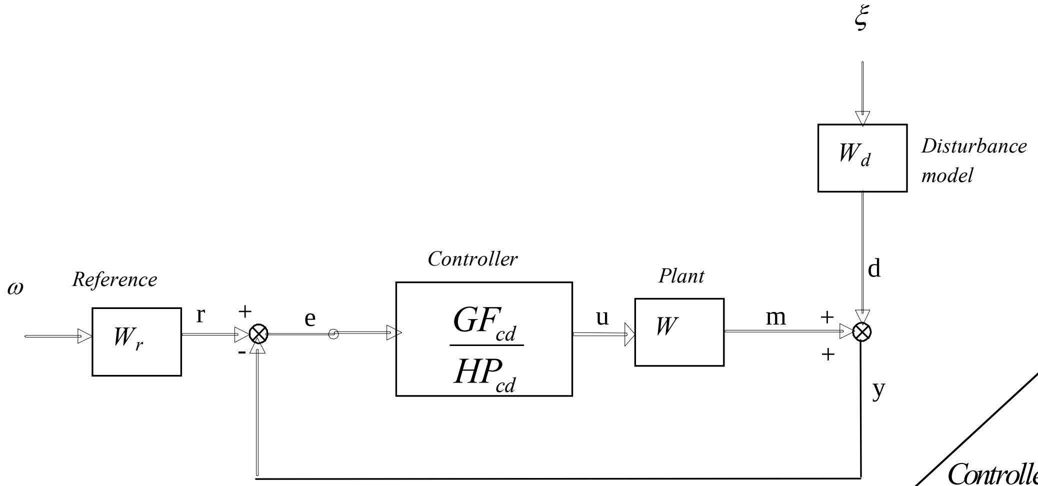
statistically independent terms

Optimal GMV control: $u^{GMV}(t) = -\dfrac{R}{(P_cW - F_c)F}\phi_0(t)$

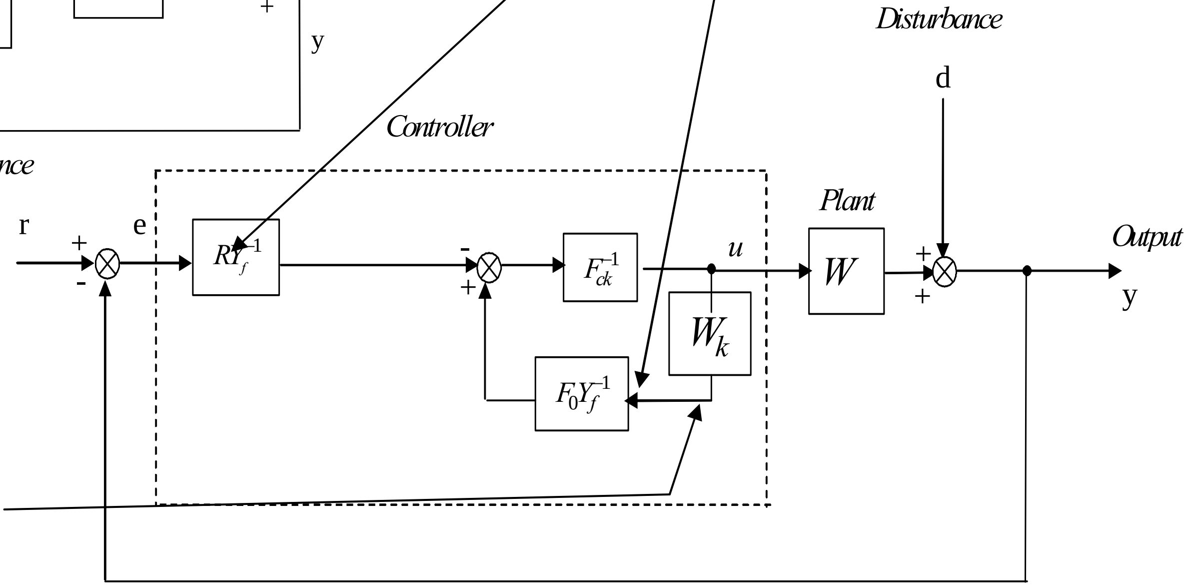
Polynomial solution:

$$u^{GMV}(t) = -\frac{R}{(F_{ck} - FY_f^{-1}W_k)Y_f}e(t) = \frac{GF_{cd}}{HP_{cd}}e(t)$$

GMV Controller Implementation



Subsystems depending only on the noise models, error weighting and the time delay *k*



Delay-free plant model

- Introduction
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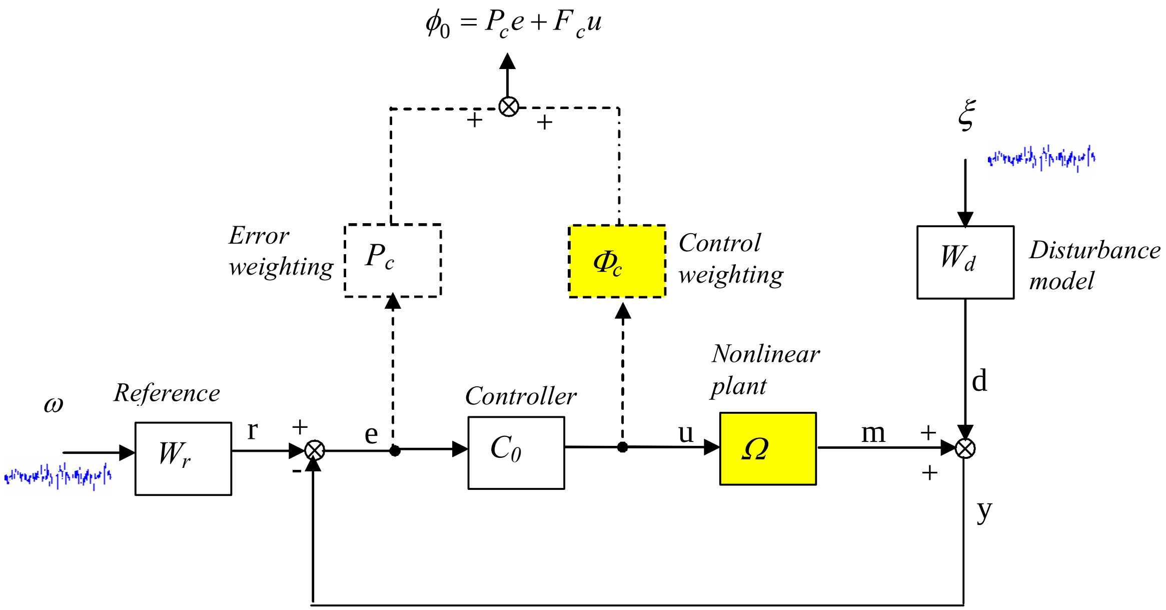
Nonlinear System Description



Nonlinear plant model: $\left(\mathrm{W}u\right)(t) = z^{-k}\left(\mathrm{W}_k u\right)(t)$

Disturbance model: *(assumed linear)* $W_d = A_f^{-1} C_d$

Reference model: *(assumed linear)* $W_r = A_f^{-1} E_r$

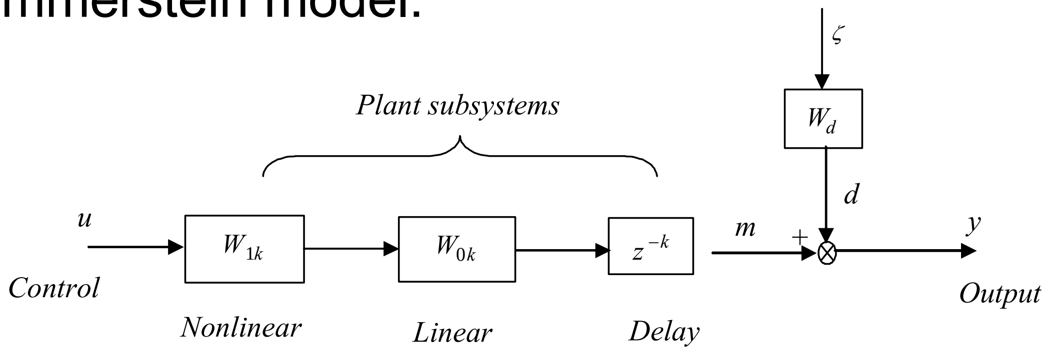
Plant Model

Nonlinear plant model may be given in a very general form, e.g.:

- state-space formulation
- neural network / neuro-fuzzy model
- look-up table
- Fortran/C code

$$f(u, y) = 0$$

It can include both linear and nonlinear components, e.g. Hammerstein model:



Just need to obtain the output to given input signal

Nonlinear GMV Problem Formulation

The cost function is as in the linear case:

$$J_{NGMV} = E[\phi_0^2(t)]$$

with $\phi_0(t) = P_c e(t) + (F_c u)(t)$

$P_c = \dfrac{P_{cn}}{P_{cd}}$ - linear error weighting

$(F_c u)(t) = z^{-k}(F_{ck} u)(t)$ - possibly nonlinear control weighting

- Control weighting invertible and potentially nonlinear to compensate for plant nonlinearities
- The weighting selection is restricted

Nonlinear GMV Problem Solution

The approach also similar:

$$\phi(t) = P_c(-z^{-k}(\mathrm{W}_k u)(t) + Y_f \varepsilon(t)) + (\mathrm{F}_c u)(t)$$

$$= z^{-k}(\mathrm{F}_{ck} - P_c \mathrm{W}_k)u(t) + P_c Y_f \varepsilon(t)$$

$$P_c Y_f = F + z^{-k} R$$

Diophantine equation

$$\phi(t) = F\varepsilon(t) + \left[(\mathrm{F}_{ck} - P_c \mathrm{W}_k)u(t-k) + R\varepsilon(t-k) \right]$$

statistically independent

$\varepsilon(t)$ – white noise (sequence of independent random variables)

Optimal control: $$u^{NGMV}(t) = -(\mathrm{F}_{ck} - P_c \mathrm{W}_k)^{-1} R\varepsilon(t)$$

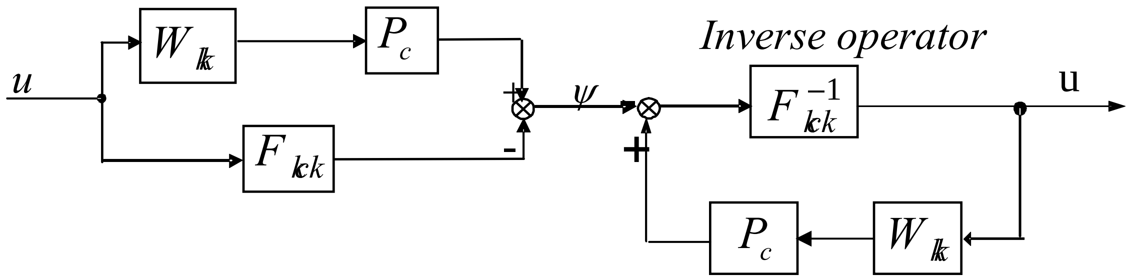
stable causal nonlinear operator inverse

Nonlinear Operator and its Inverse

By definition:

$$\left(P_c W_k - F_{ck}\right)u = P_c\left(W_k u\right)(t) - \left(F_{ck} u\right)(t) = \psi(t)$$

$$u(t) = F_{ck}^{-1}\left[P_c\left(W_k u\right)(t) - \psi(t)\right]$$



- Control weighting assumed invertible
- For the closed-loop stability, the nonlinear operator must be invertible in the operating region
- Problem: algebraic loop

Algebraic Loop

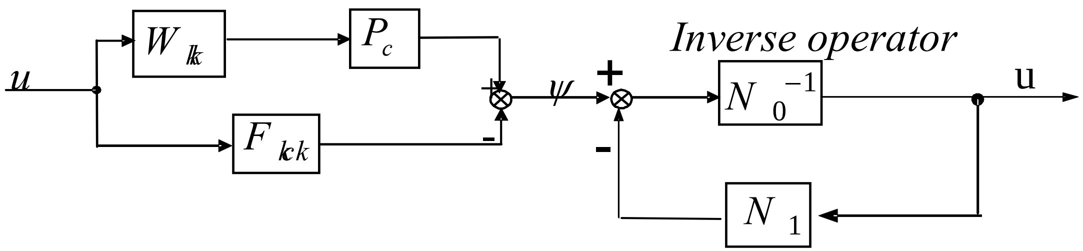
Problem solutions:

- solve the loop iteratively on-line
- introduce an additional delay in the loop
- transform into equivalent problem:

→ split the nonlinear operator into two parts involving a delay-free term N_0 and a term that depends upon past values of the control action N_1:

$$\psi(t) = (P_c W_k - F_{ck})u = (N_0 u)(t) + (N_1 u)(t)$$

$$u(t) = N_0^{-1}\left(\psi(t) - (N_1 u)(t)\right)$$

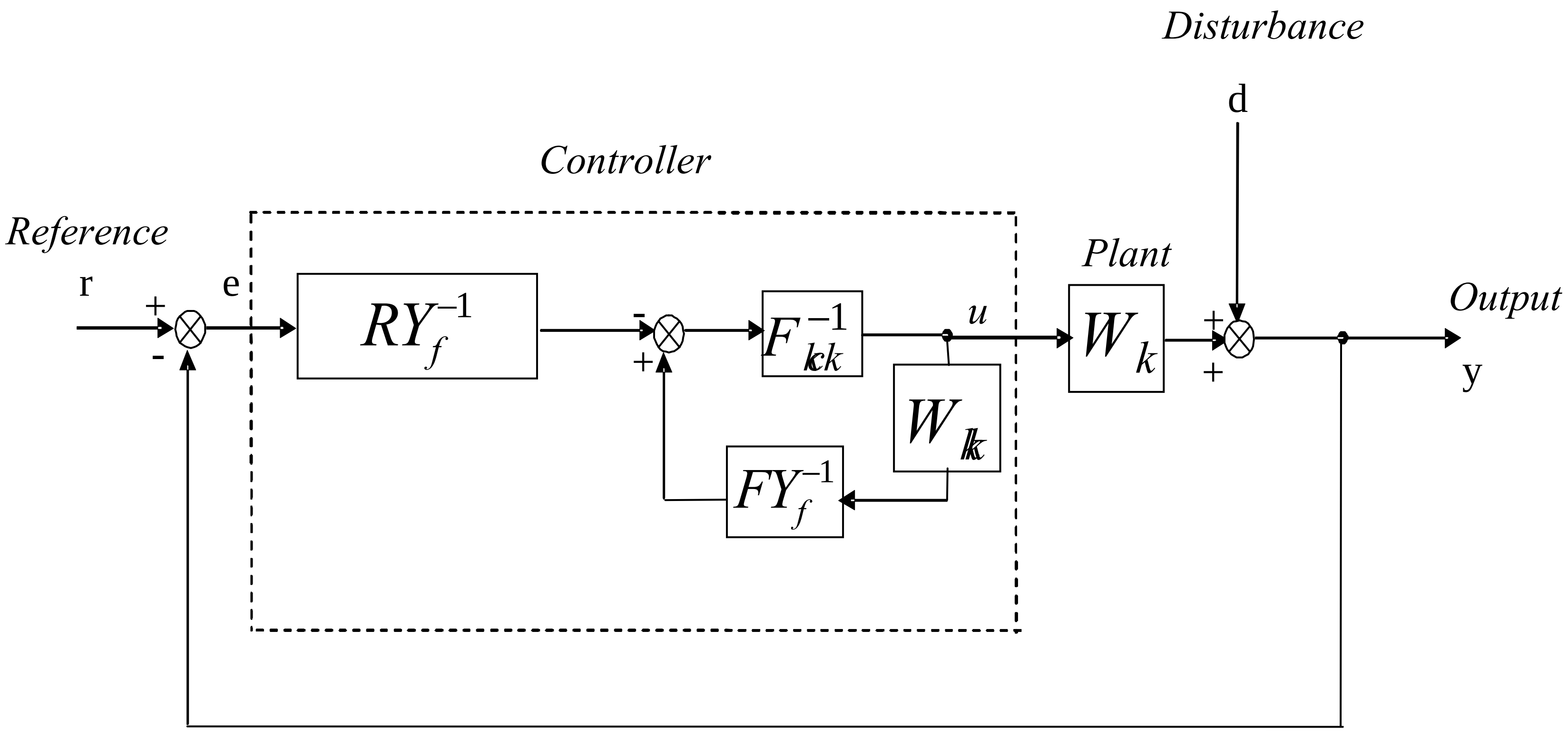


$$N_0 = N\Big|_{z^{-1}=0} = (P_c W_k - F_{ck})\Big|_{z^{-1}=0}$$

$$N_1 = N - N_0 .$$

Controller Implementation

$$u^{NGMV}(t) = -[(\mathrm{F}_{ck} - FY_f^{-1}\mathrm{W}_k)^{-1} RY_f^{-1} e](t)$$



Bias and Steady-State Levels

- So far the assumption was on zero mean exogenous signals
- Behaviour at an operating point of interest

Signal notation: $x = \bar{x} + \tilde{x}$

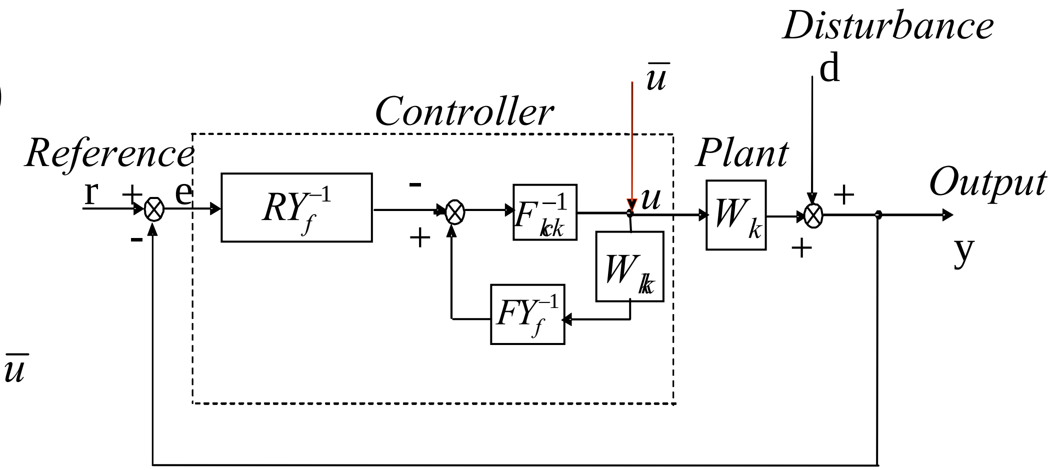
bias — deviation

Modified signal $\phi_0(t)$:

$$\phi_0(t) = P_c e(t) + (F_c u)(t) - (F_c \bar{u})(t)$$

Re-derived control:

$$u^{opt}(t) = -[(F_{ck} - FY_f^{-1} W_k)^{-1} RY_f^{-1} e](t) + \bar{u}$$



Design of the Weightings

- Restriction on the choice of weightings: invertibility of the nonlinear operator

$$\left(P_c W_k - F_{ck}\right)$$

- Control weighting may be used to control a non-invertible plant

- Admissible and meaningful choice of the weightings is the subject of current research

- Adaptation of the linear-case rules of thumb:
    - P_c normally high at low frequencies to guarantee integral action
    - F_c high at large frequencies to provide sufficient controller roll-off
    - properties close to those of LQG-type controllers
    - closed-loop bandwidth normally close to cross-over frequency of the open-loop system → loop shaping

Design of the Weightings (cont.)

Consider Φ_{ck} linear and negative: $F_{ck} = -F_k$

Then $(P_c W_k + F_k)u = F_k \left(1 + \frac{P_c}{F_k} W_k \right) u$

return-difference operator for a feedback system with the delay-free plant and controller $\frac{P_c}{F_k}$

Consider the delay-free plant Ω_k and assume a PID controller K_{PID} exists to stabilize the closed-loop system.

Then a starting point for the weighting choice that will ensure the operator $(P_c W_k + F_k)$ is stably invertible is

$$\frac{P_c}{F_k} = K_{PID}$$

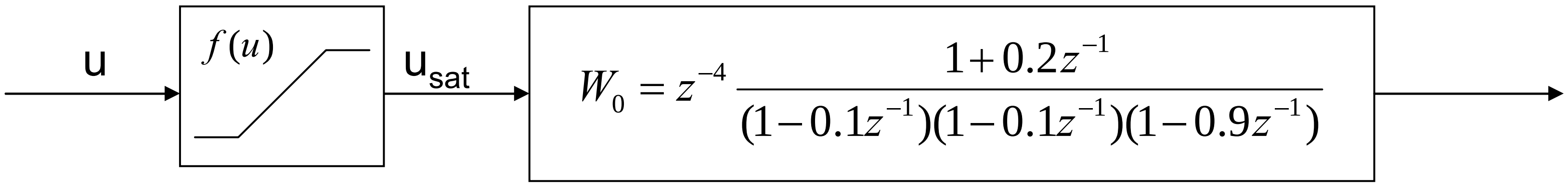
Special Case: Nonlinear MV Controller

Note that in the limiting case, for a square system, when $F_{ck} \to 0$ the optimal control signal becomes

$$u_{\mathrm{mv}}(t) = (P_c W_k)^{-1} R Y_f^{-1}(r(t) - d(t))$$

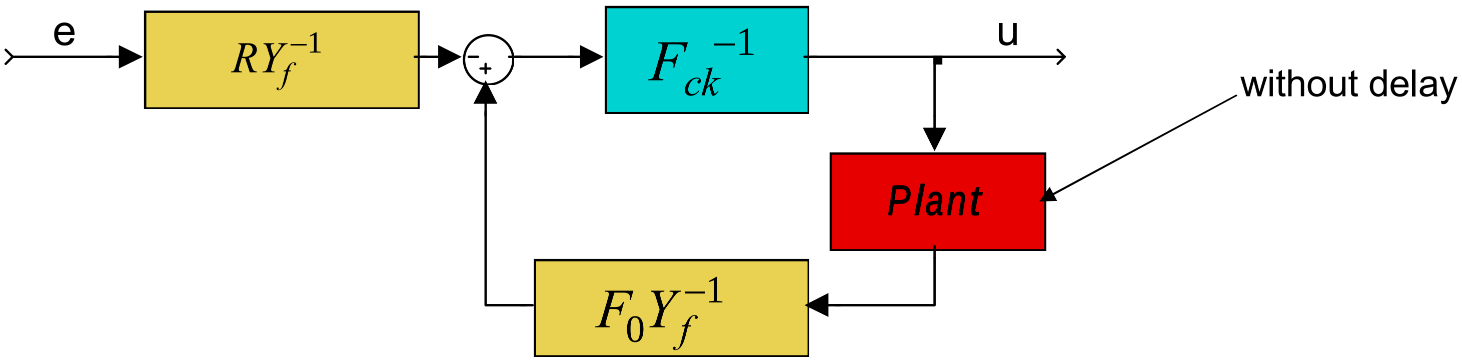
Clearly the *minimum variance* control for the nonlinear system includes the stable inverse of the plant model, when one exists.

Special Case: Actuator Saturation



Problem: select <u>invertible</u> control weighting Φ_{ck} such that an anti-windup mechanism is achieved

Nonlinear GMV Controller



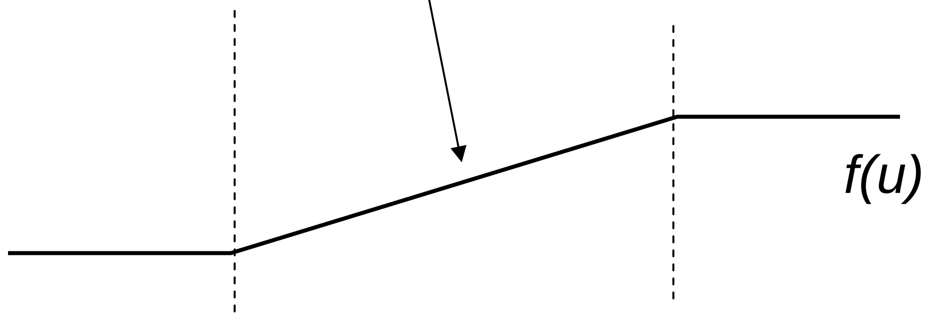
Actuator Saturation (cont.)

Control weighting choice:

$$(F_{ck}u)(t) = (\hat{F}_{ck}u)(t) + \frac{\rho}{1-z^{-1}}\left[u - f(u)\right]$$

nominal weighting

integrator, becomes active at saturation

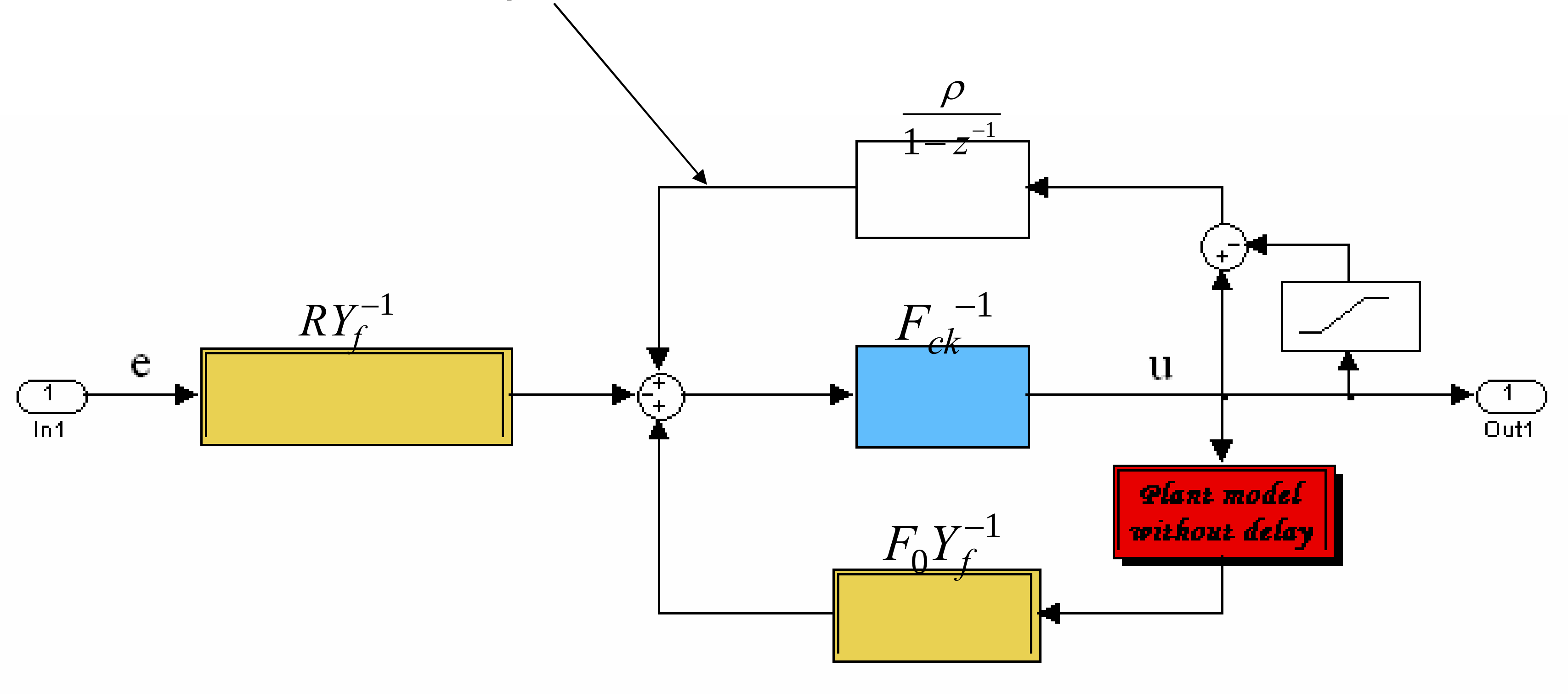


Control signal becomes:

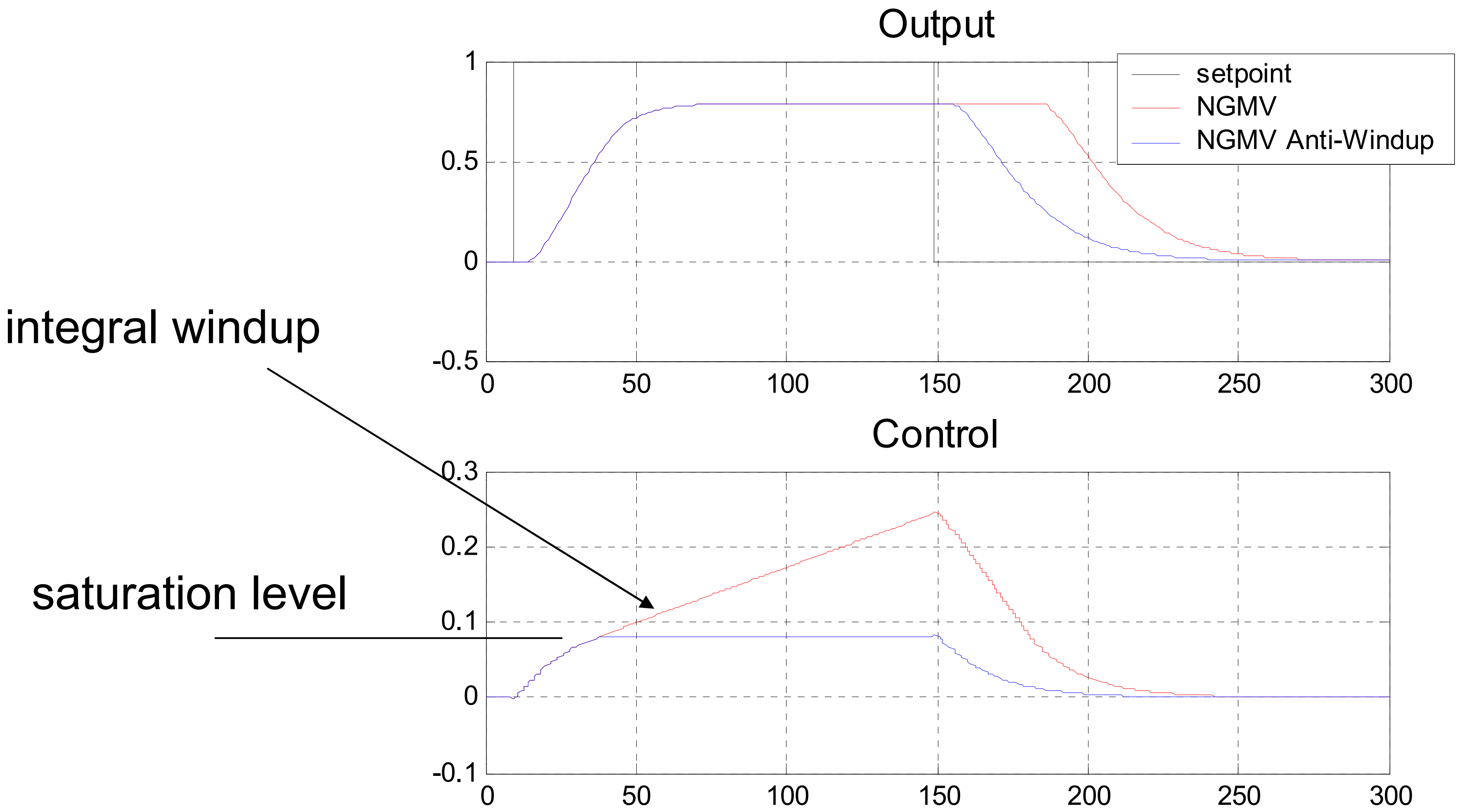
$$\bar{u}(t) = F_{ck}^{-1}\left(F_0 Y_f^{-1}\left(W_k u\right)\left(t\right) - R Y_f^{-1} \underset{\sim}{e}\left(t\right) - \frac{\rho}{1-z^{-1}}(u - f(u)) \right)$$

Actuator Saturation – Block Diagram

anti-windup scheme



Actuator Saturation - Example

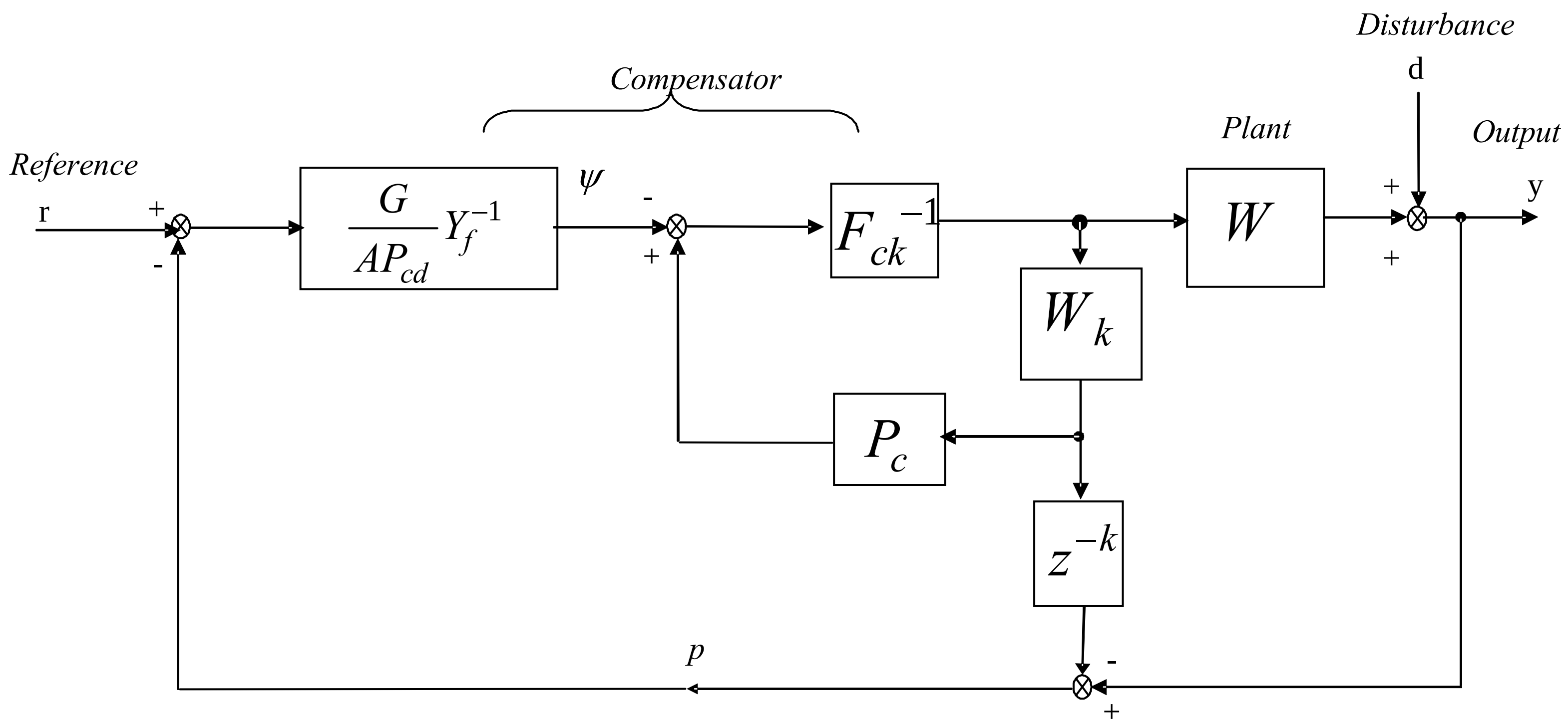


Nonlinear Smith Predictor

- Optimal NGMV controller can be expressed in a similar form to that of a Smith predictor
- Introduction of this structure limits the application of the solution on open-loop unstable systems
- The structure is intuitively reasonable and should be valuable in applications
- The Smith predictor results by rearranging the controller structure

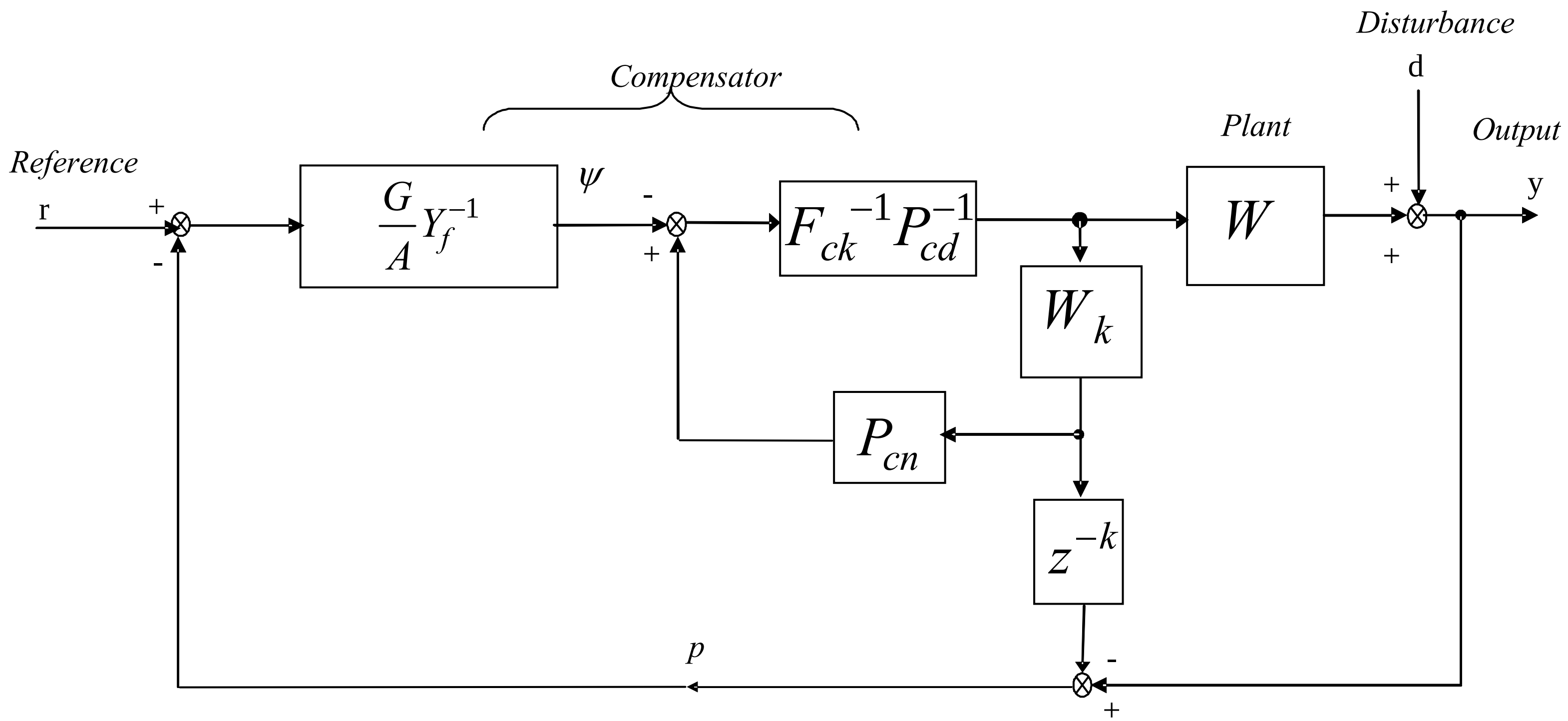
Nonlinear Smith Predictor

The control loop can be rearranged as follows:



Nonlinear Smith Predictor

When the error weighting P_c includes an integrator, it must be placed in the inner error channel:



Extensions

- NGMV Feedback, Feedforward and Tracking control
- Multivariable version → time-delay matrix
- Modelling issues – "Neuro-fuzzy NGMV control"
- State-space representation

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Estimation of the Minimum Variance

NGMV controller cancels the (generalized) plant dynamics and the generalized output signal $\phi_0(t)$ is a moving-average time series:

$$\phi_0^{ngmv}(t) = F\varepsilon(t) = f_0\varepsilon(t) + f_1\varepsilon(t-1) + \ldots + f_{k-1}\varepsilon(t-k+1)$$

The minimum variance (the benchmark) follows as

$$J_{\min} = Var\left[F\varepsilon(t)\right] = \sum_{i=0}^{k-1} f_i^2$$

This value depends only on the noise model and the plant time delay.

Problem: estimate J_{min} from the collected closed-loop data.

"Harris Algorithm"

Harris (1989) and *Desborough and Harris* (1992, 1993):

- model the controller-dependent part of the output as AR time series
- estimate the minimum variance as the residual error variance

The generalized algorithm applies to the signal $\phi_0(t)$ rather than $y(t)$:

$$\phi_0(t) = F\varepsilon(t) + \sum_{i=0}^{m} \alpha_i \phi_0(t-k-i)$$

Collect N samples of data and write in matrix form:

$$\bar{\phi} = X\bar{\alpha} + \bar{\varepsilon} \quad \longrightarrow \quad \bar{\alpha} = (X^T X)^{-1} X^T \bar{\phi}$$

least-squares solution

Minimum variance estimate:

$$\hat{\sigma}_{mv}^2 = \frac{1}{N-k-2m+1} (\bar{\phi} - X\bar{\alpha})^T (\bar{\phi} - X\bar{\alpha})$$

FCOR Algorithm

Huang and Shah (1999): Filtering and Correlation algorithm

- model the output as an AR time series and estimate the white noise generating sequence (*innovations sequence*)
- correlate the obtained white noise with the output

As applied to the generalized signal $\phi_0(t)$:

Whitening process:

$$\phi_t = \frac{1}{A(q^{-1})}\varepsilon_t \quad \Rightarrow \quad \varepsilon_t = A(q^{-1})\phi_t$$

Cross-correlation:

$$r_{\phi\varepsilon}(0) = E[\phi_t \varepsilon_t] = f_0$$

$$r_{\phi\varepsilon}(1) = E[\phi_t \varepsilon_{t-1}] = f_1$$

$$\vdots$$

$$r_{\phi\varepsilon}(k-1) = E[\phi_t \varepsilon_{t-k+1}] = f_{k-1}$$

Minimum variance:

$$J_{\min} = \sum_{i=0}^{k-1} f_i^2$$

Controller Performance Index

For the existing controller

$$J = Var[\phi_0] \geq J_{\min}$$

Definition of the Controller Peformance Index:

$$\kappa = \frac{J_{\min}}{J}$$

1 (NGMV control)

0 (very poor control)

"Harris index"

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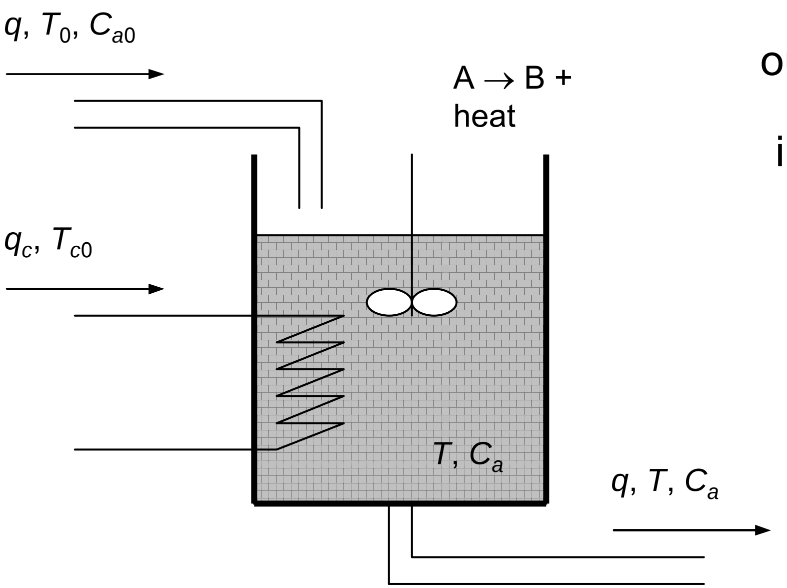
Continuous Stirred Tank Reactor



output → Product concentration

input → Coolant flow rate

	Symbol	unit	nominal value
Product concentration	C_a	**mol/l**	**0.1**
Reactor temperature	T	K	438.54
Coolant flow rate	q_c	**l/min**	**103.41**
Process flow rate	q	l/min	100
Feed concentration	C_{a0}	mol/l	1
Feed temperature	T_0	K	350
Inlet coolant temperature	T_{c0}	K	350
CSTR volume	V	l	100
Heat transfer term	hA	cal/min/K	$7 \cdot 10^5$
Reaction rate constant	k_0	min^{-1}	$7.2 \cdot 10^{10}$
Activation energy term	E/R	K^{-1}	$1 \cdot 10^4$
Heat of reaction	ΔH	cal/mol	$-2 \cdot 10^5$
Feed density	ρ	g/l	$1 \cdot 10^3$
Coolant density	ρ_c	g/l	$1 \cdot 10^3$
Feed specific heat	C_p	cal/g/K	1
Coolant specific heat	C_{pc}	cal/g/K	1

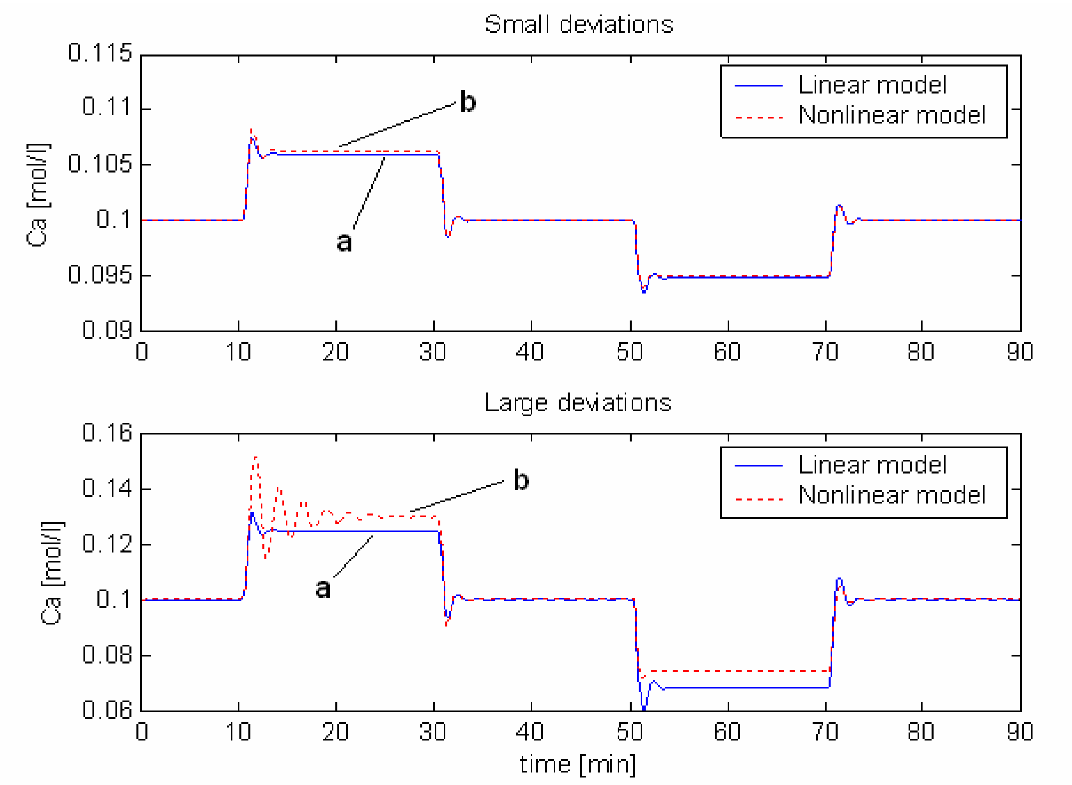
Energy balance equation

$$\dot{T}(t) = \frac{q(t)}{V}\left(T_0 - T(t)\right) + \frac{\Delta H}{\rho C_p} k_0 C_a(t) \exp\left(-\frac{E/R}{T(t)}\right) + \frac{\rho_c C_{pc}}{\rho C_p V} q_c(t)\left[1 - \exp\left(\frac{-hA}{\rho_c C_{pc} q_c(t)}\right)\right]\left(T_{c0} - T(t)\right)$$

Material balance equation

$$\dot{C}_a(t) = \frac{q(t)}{V}\left(C_{a0} - C_a(t)\right) - k_0 C_a(t) \exp\left(-\frac{E/R}{T(t)}\right)$$

Open-Loop Step Responses



(a) Linear model
(b) Nonlinear model

Linearised model $$W_{lin} = z^{-5}\frac{10^{-4}(0.01571 + 1.863z^{-1} + 0.4614z^{-2} - 1.14z^{-3})}{1 - 2.338z^{-1} + 1.88z^{-2} - 0.5098z^{-3}}$$

Optimal Controller Design

Disturbance model:

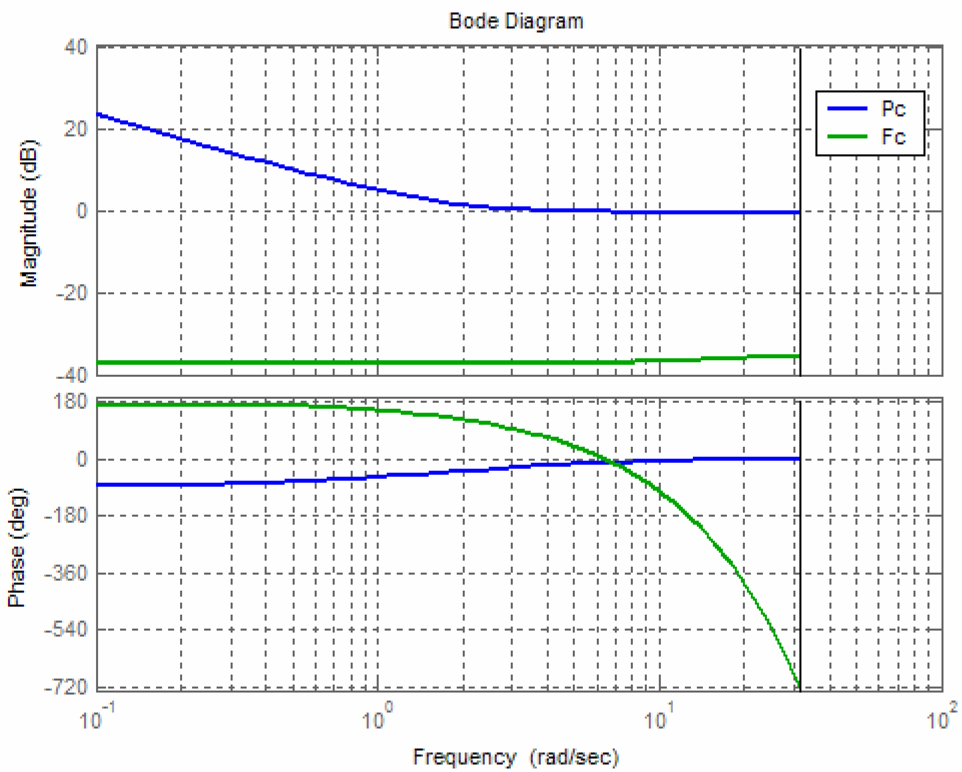
$$W_d = \frac{0.001}{1-0.95z^{-1}}$$

Dynamic weightings:

$$P_c = \frac{1-0.85z^{-1}}{1-z^{-1}}$$

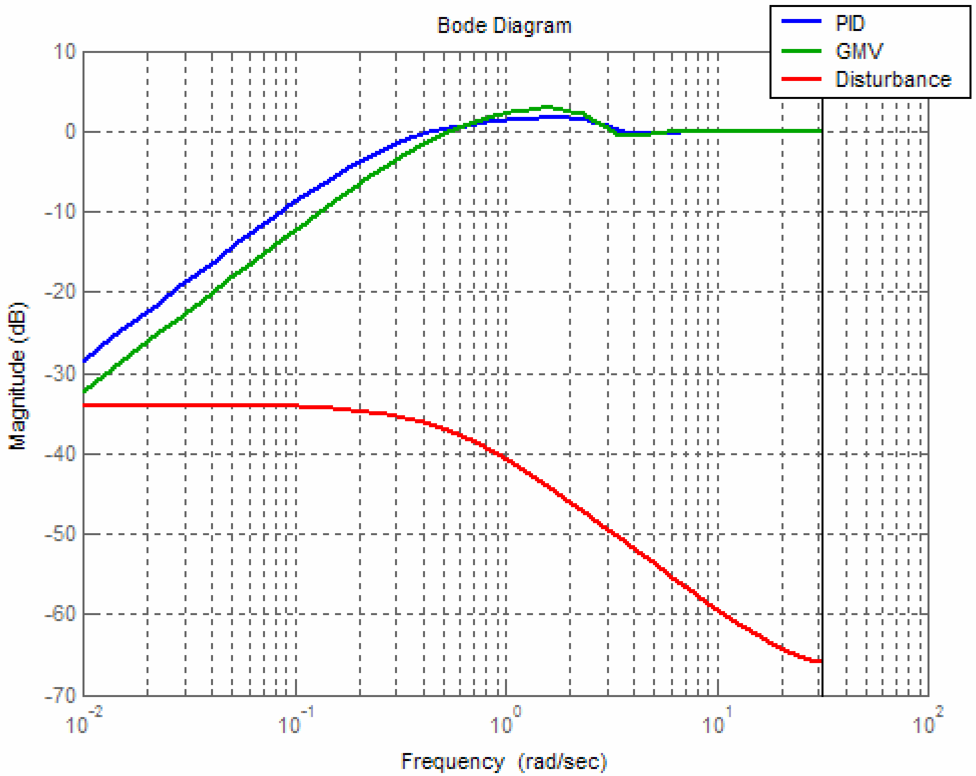
$$F_{ck} = -0.015(1-0.1z^{-1})$$

Frequency response of the weightings



GMV Controller Design

Sensitivity function plots for the PID and the linear GMV controller



Stochastic Performance

C_a [mol/l]	Controller	Var[C_a]	Var[q_c]	Var[ϕ]
0.06	PI	9.687e-6	5.789e-1	5.088e-1
	GMV	8.212e-6	7.575e-1	7.886e-2
	NGMV	8.079e-6	7.780e-1	7.323e-2
0.1	PI	9.765e-6	2.683e-1	3.198e-1
	GMV	7.495e-6	3.496e-1	7.315e-2
	NGMV	7.487e-6	3.498e-1	7.318e-2
0.12	PI	8.770e-6	2.083e-1	2.647e-1
	GMV	2.386e-5	3.418e-1	2.375e-1
	NGMV	7.536e-6	2.591e-1	7.229e-2

Benchmarking Results

Benchmarking results (nominal operating point)

Controller	Method	Jmin	J	eta
PI	Harris	7.301e-2	3.202e-1	0.228
	FCOR	7.253e-2	3.201e-1	0.227
GMV	Harris	7.329e-2	7.320e-2	1.001
	FCOR	7.304e-2	7.319e-2	0.998
NGMV	Harris	7.336e-2	7.323e-2	1.002
	FCOR	7.310e-2	7.322e-2	0.998

Benchmarking results (operating point C_a=0.06 mol/l)

Controller	Method	Jmin	J	eta
PI	Harris	7.339e-2	5.105e-1	0.144
	FCOR	7.296e-2	5.104e-1	0.143
GMV	Harris	7.340e-2	7.896e-2	0.930
	FCOR	7.315e-2	7.895e-2	0.927
NGMV	Harris	7.339e-2	7.330e-2	1.001
	FCOR	7.316e-2	7.329e-2	0.998

Benchmarking results (operating point C_a=0.12 mol/l)

Controller	Method	Jmin	J	eta
PI	Harris	7.185e-2	2.665e-1	0.270
	FCOR	7.159e-2	2.665e-1	0.269
GMV	Harris	7.966e-2	2.386e-1	0.334
	FCOR	7.949e-2	2.386e-1	0.333
NGMV	Harris	7.213e-2	7.242e-2	0.996
	FCOR	7.204e-2	7.241e-2	0.995

Comments on the Results

• NGMV shows best performance over the whole operating range

• While the linear GMV control adequate for small deviations from the nominal operating point, its performance degrades when away from it

• Overall performance of the existing PI controller remains relatively unchanged compared with the linear GMV controller – this is due to greater robustness of this simple controller

• Transient performance comparable; NGMV more robust than linear GMV away from the operating point

- Introduction
- Review of the linear GMV control theory
- Nonlinear GMV controller
- Performance assessment against NGMV controller
- Simulation example
- **Summary**

Summary

- Simple nonlinear control algorithm introduced
- Generalization of the established linear GMV controller
- Anti-windup mechanism easily incorporated
- Can be represented in the “nonlinear Smith predictor” form
- Candidate for a benchmark controller