New Developments in Performance Assessment and Benchmarking

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Introduction

Multivariable controller design and benchmarking

Restricted-structure benchmarking in MIMO case

Structure assessment

Simulation example

Summary
Introduction: MIMO Benchmarking

- Minimum variance benchmark and “Harris index” have become widespread over the last decade
- MV control rarely used but MV benchmark still useful
- GMV controller proposed as an alternative benchmark
- So far focus mostly on SISO performance
- But most processes involve interactions between a number of control loops? MIMO benchmark better
Benchmarking against optimal controllers does not take into account the existing controller structure.

Restricted-structure controller approach: compare against the best actually achievable controller.

MIMO RS approach may have other useful applications such as optimal I/O pairing and structure assessment.
• Introduction
• **Multivariable controller design and benchmarking**
• Restricted-structure benchmarking in MIMO case
• Structure assessment
• Simulation example
• Summary
Multivariable Control System Description

$\zeta_t$, $\xi_t$ - white noise vectors

System matrix fractions

$W(z^{-1}) = A^{-1}(z^{-1})B(z^{-1})$

$W_r(z^{-1}) = A^{-1}(z^{-1})E_r(z^{-1})$

$W_d(z^{-1}) = A^{-1}(z^{-1})C_d(z^{-1})$

$e_t = r_t - y_t = S_r(z^{-1})(r_t - d_t)$ - control error

$u_t = C_0(z^{-1})S_r(z^{-1})(r_t - d_t)$ - control signal

$S_r = (I_r + WC_0)^{-1}$

sensitivity function
Plant Delay Structure

Interactor matrix (IM) – generalization of the scalar time delay
Fundamental performance limitation in the system

<table>
<thead>
<tr>
<th>SISO</th>
<th>MIMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(t) = T(z^{-1})u(t) = z^{-k} \tilde{T}(z^{-1})u(t) )</td>
<td>( Y(t) = T(z^{-1})Y(t) = D^{-1}\tilde{T}(z^{-1})U(t) )</td>
</tr>
<tr>
<td>( \lim_{z^{-1} \to 0} z^{k} \cdot T(z^{-1}) = k, \quad k \neq 0 )</td>
<td>( \lim_{z^{-1} \to 0} D(z) \cdot T(z^{-1}) = K, \quad K ) finite and full rank</td>
</tr>
<tr>
<td>( \det{D(z)} = z^{k}, \quad k - number of infinite zeros )</td>
<td></td>
</tr>
</tbody>
</table>

- Interactor matrix characterizes the infinite zeros of the system
- It is often diagonal, but generally it may be a full matrix
- Interactor matrix of the system is not unique:
  - lower triangular IM (Wolovich and Falb 1976)
  - nilpotent IM (Rogozinski et al. 1987)
  - unitary IM (Peng and Kinnaert 1992)
### Performance Criteria

**GMV cost function:**

\[ J_{GMV} = \text{Var}\{\phi_t\}, \quad \phi_t = P_c e_t + F_c u_t \]

**LQG cost function:**

\[ J_{LQG} = \frac{1}{2\pi j} \oint_{|z|=1} \text{trace}\{Q_c \Phi_{ee} + R_c \Phi_{uu}\} \frac{dz}{z} \]

<table>
<thead>
<tr>
<th></th>
<th>GMV</th>
<th>LQG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design</strong></td>
<td>+ simpler</td>
<td>+ guaranteed stability</td>
</tr>
<tr>
<td></td>
<td>- restriction on the weightings</td>
<td>- more involved</td>
</tr>
<tr>
<td><strong>Benchmarking</strong></td>
<td>+ data driven</td>
<td>+ “classical” optimal benchmark</td>
</tr>
<tr>
<td></td>
<td>- interactor matrix needs to be known or estimated</td>
<td>- full model needed</td>
</tr>
</tbody>
</table>
GMV Control: Generalized Plant

Minimum Variance Control:
\[ y_t = D^{-1} \tilde{T} u_t + N \xi_t \quad \Rightarrow \quad \tilde{\phi}_t = q^{-k} D y_t = q^{-k} \tilde{T} u_t + \underbrace{q^{-k} D N \xi_t}_{\tilde{N}} \]

Generalized Minimum Variance Control:
\[ \phi_t = P_c e_t + F_c u_t = P_c (D^{-1} \tilde{T} u_t + N \xi_t) + F_c u_t = (P_c D^{-1} \tilde{T} + F_c) u_t + P_c N \xi_t = \]
\[ = D^{-1}_g \tilde{T}_g u_t + N_g \xi_t \quad D_g \text{ unitary} \]

Interactor filtering:
\[ \tilde{\phi}_t = q^{-k} D_g \phi_t = q^{-k} \tilde{T}_g u_t + N_g \xi_t \]

\[ \text{Var} \left\{ \phi_t \right\} = \text{Var} \left\{ \tilde{\phi}_t \right\} \]
Multivariable GMV Controller

Main result:

\[ J = J_{\text{min}} + J_0 \]

\[ J_{\text{min}} = \frac{1}{2\pi j} \oint \text{trace}\{F^*F\} \frac{dz}{z} \]

minimum achievable value

\[ J_0 = \frac{1}{2\pi j} \oint \text{trace}\{X^*X\} \frac{dz}{z} \]

suboptimal term

\[ X = (GD_2^{-1}P_{cd}^{-1}C_{0d} - HD_3^{-1}F_{cd}^{-1}C_{0n})(AC_{0d} + BC_{0n})^{-1}D_f \]

Diophantine equations

\[ D_f^{-1} A_{P_{cd}} = A_2 D_2^{-1} \quad FA_2 + z^{-k} G = P_{cn} D_2 \]

\[ D_f^{-1} B_{F_{cd}} = B_2 D_3^{-1} \quad FB_2 - z^{-k} H = F_{cn} D_3 \]

filter spectral factor

\[ C_0 = F_{cd} D_3 H^{-1} G D_2^{-1} P_{cd}^{-1} \]

optimal controller
MIMO GMV Benchmarking

Benchmarking procedure – FCOR algorithm (Huang and Shah 1999)

\[ \varepsilon_t = A(z^{-1}) \phi_t \iff \phi_t = A^{-1}(z^{-1}) \varepsilon_t \]

whitening

\[ \phi_t = P_c e_t + F_c u_t \]

filtering

\[ E[\phi_{t-j} \varepsilon_t] = F_j, \quad j = 0, \ldots, k-1 \]

correlation

\[ J_{\text{min}} \]

\[ \eta = \frac{J_{\text{min}}}{\text{Var}[\phi_t]} \]
• Introduction
• Multivariable controller design and benchmarking
• Restricted-structure controllers
• Structure assessment
• Simulation example
• Summary
Motivation

- Often a need for relatively unskilled staff to tune controllers and this implies low order controllers needed.
- Low order controllers often have improved robustness properties relative to high order designs.
- Often a requirement to have the good control action of advanced designs within a controller structure that is simple.
- Nonlinear systems can be linearised at operating points and multiple model RS controllers designed.
- Optimal RS controllers can be used as realistic benchmarks; however, model needed in this case.
Reduced Controller Structures

<table>
<thead>
<tr>
<th>Reduced order:</th>
<th>$C_0(s) = \frac{a_n s^n + \ldots + a_0}{s^n + \ldots + a_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead lag:</td>
<td>$C_0(s) = \frac{(a_n s^n + \ldots + a_0)(c_2 s^2 + c_1 s + c_0)}{(b_n s^n + \ldots + b_0)(d_2 s^2 + d_1 s + d_0)}$</td>
</tr>
<tr>
<td>PID:</td>
<td>$C_0(s) = k_0 + k_1 / s + k_2 s$</td>
</tr>
<tr>
<td>Filtered PID:</td>
<td>$C_0(s) = k_0 + \frac{k_1}{s} + \frac{k_2 s}{\tau s + 1}$</td>
</tr>
</tbody>
</table>

where $n = p$ is less than the order of the system (plus weightings)

Swing modes:

<table>
<thead>
<tr>
<th>Diagonal:</th>
<th>$\begin{bmatrix} C_0 \ C_1 \ C_2 \end{bmatrix}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular:</td>
<td>$\begin{bmatrix} C_0 &amp; C_{01} &amp; C_{02} \ C_1 &amp; C_{12} \ C_2 \end{bmatrix}$</td>
</tr>
<tr>
<td>Full:</td>
<td>$\begin{bmatrix} C_{11} &amp; C_{12} &amp; C_{13} \ C_{21} &amp; C_{22} &amp; C_{23} \ C_{31} &amp; C_{32} &amp; C_{33} \end{bmatrix}$</td>
</tr>
</tbody>
</table>

The assumption must be made that a stabilising control law exists for the assumed controller structure.
Restricted-Structure Design Issues

• MIMO structure: choice of input/output variables, I/O pairing, additional couplings (can use the existing structure)
• Restricted controller structure for each element of the transfer-function matrix
• Optimality criterion - e.g. GMV, LQG, $H_\infty$ cost functions
• Optimization algorithm (how to find the “optimal” controller parameters)
RS-LQG Control – SISO Case

Stochastic System Description

\[ e = r - y \quad u = C_0 S(r - d) \]

\[ \xi, \zeta \quad - \text{independent white noise sequences} \]

LQG cost function to minimize:

\[ J = \frac{1}{2\pi j} \oint_{|z|=1} \left\{ Q_e \Phi_{ee} + R_c \Phi_{uu} \right\} \frac{dz}{z} \]

\[ W(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} \]

\[ W_d(z^{-1}) = \frac{C_d(z^{-1})}{A(z^{-1})} \]

\[ W_r(z^{-1}) = \frac{E_r(z^{-1})}{A(z^{-1})} \]

\[ S = \frac{1}{1 + WC_0} \]

sensitivity function

Dynamic weightings

\[ Q_c = H_q^* H_q = \frac{B_q^* B_q}{A_q^* A_q} \]

\[ R_c = H_r^* H_r = \frac{B_r^* B_r}{A_r^* A_r} \]
**Step 1**: Design full-order optimal controller

- Introduce an innovations model for the signal:

\[ D_f D_f^* = E_r E_r^* + C_d C_d^* \]

- noise spectral factor

\[ e = S Y_f \varepsilon \quad \text{and} \quad u = C_0 S Y_f \varepsilon \]

\[ D_c D_c^* = B^* A_r^* B_q^* B_q A_r B + A^* A_q^* B_r^* B_r A_q A \]

Spectral factorization

\[ \bar{D}_c G + FAA_q = \overline{BA}_q Q_n D_f \]

\[ \bar{D}_c H - FBA_r = \overline{AA}_q R_n D_f \]

Diophantine equations

Suboptimal component:

\[ J = J_{\text{min}} + J_0 \]

\[ T_0 = \frac{H A_q C_{0n} - G A_r C_{0d}}{A_q A_r (A C_{0d} + B C_{0n})} \]
RS-LQG Control – SISO Case

Step 2: Restricted-structure optimization problem

The optimal controller must be chosen in such a way that \( J_0 \) is minimized:

\[
\text{Min} \quad J_0 \quad \{k_0, k_1, k_2\}
\]

\[
T_0 = \frac{HA_qC_{0n} - GA_rC_{0d}}{A_qA_r(AC_{0d} + BC_{0n})}
\]

subject to the controller structure:

\[
C_0(z^{-1}) = k_0 + \frac{k_1}{1-z^{-1}} + \frac{k_2(1-z^{-1})}{1- \alpha z^{-1}}
\]

\[
J_0 = \frac{1}{2\pi j} \oint_{|z|=1} T_0(z^{-1})T_0^*(z)z^{-1} dz = \frac{T_s}{2\pi} \int_0^{2\pi/T_s} T_0(e^{j\omega T_s})T_0(e^{-j\omega T_s}) d\omega
\]

\[
\approx \frac{T_s}{2\pi} \sum_{i=1}^{N} T_0(e^{j\omega_i T_s})T_0(e^{-j\omega_i T_s}) \cdot \Delta \omega_i
\]
Optimization

Min \[ J_0 \]
\[ \{k_0, k_1, k_2\} \]

- This is a nonlinear optimization problem and generally does not have a closed analytical solution
- Iterative numerical solution required
  - Successive approximation / Edmunds’ algorithm
  - Gradient methods / Optimization toolbox
  - Search algorithms (genetic, evolutionary etc.)
RS-LQG: Successive approximation

- Start: initialize $C_0$
- Calculate denominator of $T_0$
- Minimize w.r.t. $C_0$ assuming constant denominator
- Solution converged OR Maximum No. of iterations reached?
  - yes
  - Return $C_0$ and stop
  - no
  - least squares minimization

\[
\text{Minimize } \sum_{i=1}^{N} T_0(e^{j\omega_i T_s})T_0(e^{-j\omega_i T_s})\Delta\omega_i
\]

\[
T_0 = \frac{HA_q C_{0n} - GA_r C_{0d}}{A_q A_r (AC_{0d} + BC_{0n})}
\]
Minimize w.r.t. $C_0$ assuming constant denominator

$$\min_{C_0} \sum_{i=1}^{N} T_0 (e^{j\omega_i T_s}) T_0 (e^{-j\omega_i T_s}) \Delta \omega_i$$

$$T_0 = \frac{HA_q C_{0n} - GA_r C_{0d}}{A_q A_r (AC_{0d} + BC_{0n})}$$

PID Controller (linear w.r.t. parameters):

$$C_{0n}(z^{-1}) = \alpha_0(z^{-1}) k_0 + \alpha_1(z^{-1}) k_1 + \alpha_2(z^{-1}) k_2$$

$$C_{0d} = (1 - z^{-1})(1 - \alpha z^{-1})$$

- Choose the frequency range and number of frequency points $N$
- For all frequency points calculate all elements of $F$ and $L$: $T_0 = FX - L$

where:

$$F = \begin{bmatrix} f_{11}^0(\omega_1) & f_{12}^0(\omega_1) & f_{13}^0(\omega_1) \\ \vdots & \vdots & \vdots \\ f_{11}^N(\omega_N) & f_{12}^N(\omega_N) & f_{13}^N(\omega_N) \\ f_{21}^0(\omega_1) & f_{22}^0(\omega_1) & f_{23}^0(\omega_1) \\ \vdots & \vdots & \vdots \\ f_{21}^N(\omega_N) & f_{22}^N(\omega_N) & f_{23}^N(\omega_N) \\ f_{31}^0(\omega_1) & f_{32}^0(\omega_1) & f_{33}^0(\omega_1) \\ \vdots & \vdots & \vdots \\ f_{31}^N(\omega_N) & f_{32}^N(\omega_N) & f_{33}^N(\omega_N) \end{bmatrix}, \quad L = \begin{bmatrix} L_{11}^0(\omega_1) \\ \vdots \\ L_{11}^N(\omega_N) \\ L_{12}^0(\omega_1) \\ \vdots \\ L_{12}^N(\omega_N) \\ L_{13}^0(\omega_1) \\ \vdots \\ L_{13}^N(\omega_N) \end{bmatrix}, \quad x = [k_0 \ k_1 \ k_2]^T$$

- Calculate optimal $x$:

$$x^{opt} = (F^T F)^{-1} F^T L$$

RS- LQG: Successive Approximation
Multivariable RS-LQG Control Design

- Similar procedure can be applied to design the optimal RS-LQG controller in MIMO case
- The suboptimal cost function component to be minimized becomes:

$$J_0 = \frac{1}{2\pi j} \int_{|z|=1} \text{trace}\{X^* X\} \frac{dz}{z}$$

$$C_0(z^{-1}) = K_0 + \frac{K_1}{1 - z^{-1}} + \frac{K_2(1 - z^{-1})}{1 - \alpha z^{-1}}$$

where

$$X = (GD_2^{-1} A_{q}^{-1} C_{0d} - HD_3^{-1} A_{r}^{-1} C_{0n})(AC_{0d} + BC_{0n})^{-1} D_f$$

- $J_0$ is a scalar so gradient and search algorithms can still be applied to find the optimal solution
- the successive approximation algorithm can also be generalized and least squares solution used
Multivariable RS-LQG Benchmarking

The procedure:

(1) Specify the restricted structure – e.g. multi-loop with PID controllers

(2) Find the RS controller by minimizing $J_0$

(3) Calculate the minimum value of $J_0$

$$J_{0}^{\text{opt}} = \frac{1}{2\pi j} \oint_{|z|=1} \text{trace}\{X_{opt}^* X_{opt}\} \frac{dz}{z}$$

(4) Calculate this value for the existing controller structure

$$J_{0}^{\text{act}} = \frac{1}{2\pi j} \oint_{|z|=1} \text{trace}\{X_{act}^* X_{act}\} \frac{dz}{z}$$

(5) Calculate Controller Performance Index

$$K = \frac{J_{\text{min}} + J_{0}^{\text{opt}}}{J_{\text{min}} + J_{0}^{\text{act}}}$$
Multiple models

- Above algorithms valid for linear system description
- However, real systems are inherently nonlinear
- One possible solution:
  - Use a number of linear models defined for different operating points

![Diagram showing multiple linear models for different operating ranges.](image)

- Simple or weighted average
• Introduction
• Multivariable controller design and benchmarking
• Restricted-structure benchmarking in MIMO case
• **Structure assessment**
• Simulation example
• Summary
Multivariable Interactive Control System

effective process controlled by $C_2$
Issues in the Multivariable Control Design

• Selection of the variables to be controlled
• Choice of the manipulated variables
• Interactions between loops
• Self-regulatory v. non-self regulatory loops
• I/O pairing
• Joint stability regions may shrink!
• Decoupling v. multivariable control
I/O Pairing and Relative Gain Array

• Problem of the “best” choice of input/output pairs for a multi-loop control system
• Relative Gain Array (Bristol 1966) widely used in industry
• Steady-state gains required (deviations about operating point) – obtained from step tests or from the linear model
• Relative Gain Array defined as

\[ RGA = (K^{-1})^T \otimes K \]

where \( K \) is the matrix of static gains between all inputs and all outputs

element-by-element product
I/O Pairing and Relative Gain Array

• RGA close to unity = desired pairing with no interactions
• RGA close to zero = pairing should be avoided
• All elements approx. equal = strongest possible interactions
• Results might be against “conventional wisdom”
• Takes into account only steady-state information
Application of RS Design to I/O Pairing

For a 3x3 MIMO system, the following multi-loop PID control configurations are possible:

$A) \begin{bmatrix} PID & PID \\ PID \\ PID \end{bmatrix}$  $B) \begin{bmatrix} PID \\ PID \\ PID \end{bmatrix}$  $C) \begin{bmatrix} PID \\ PID \\ PID \end{bmatrix}$

$D) \begin{bmatrix} PID \\ PID \\ PID \end{bmatrix}$  $E) \begin{bmatrix} PID \\ PID \\ PID \end{bmatrix}$  $F) \begin{bmatrix} PID \\ PID \\ PID \end{bmatrix}$

• Using RS design technique it is possible to determine the optimal structure in terms of the specified cost function (takes into account also dynamic properties of the system)
• A means of optimally tuning the controllers
• Requirement: full dynamic model of the system
Application of RS Design to Structure Assessment

The controller structure is restricted on two levels, e.g.

\[
\begin{align*}
\text{Multi-loop} & : [\begin{array}{ccc}
PI & PI & PID \\
PI & & \\
& PID & \\
\end{array}] \\
\text{Triangular} & : [\begin{array}{ccc}
PI & PID & PD \\
PI & P & \\
& PID & \\
\end{array}] \\
\text{Full PID structure} & : [\begin{array}{ccc}
PID & PID & PID \\
PID & PID & PID \\
PID & PID & PID \\
\end{array}]
\end{align*}
\]

- Possible to determine potential benefits that may result from adding additional controllers to the multi-loop system
- When already decided on a particular structure, the optimal controller parameters readily available
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System Model

Example: regulatory control of a simple 2x2 system with time delays (Huang & Shah 1999)

Plant model:

\[ W = \begin{bmatrix} \frac{z^{-1}}{1-0.4z^{-1}} & \frac{K_{12}z^{-2}}{1-0.1z^{-1}} \\ \frac{0.3z^{-1}}{1-0.1z^{-1}} & \frac{z^{-2}}{1-0.8z^{-1}} \end{bmatrix} \]

Disturbance model:

\[ W_d = \begin{bmatrix} \frac{1}{1-0.5z^{-1}} & \frac{-0.6}{1-0.5z^{-1}} \\ \frac{0.5}{1-0.5z^{-1}} & \frac{1}{1-0.5z^{-1}} \end{bmatrix} \]

Parameter \( K_{12} \) determines interaction between input 2 and output 1.

The plant has a general interactor matrix \( D \):

\[ D = \begin{bmatrix} -0.9578z & -0.2873z \\ 0.2873z^2 & -0.9578z^2 \end{bmatrix} \]
Estimation of the Minimum Variance

Existing controller:

\[
C_0 = \begin{bmatrix}
0.5 - 0.2z^{-1} & 0 \\
1 - 0.5z^{-1} & 0.25 - 0.2z^{-1} \\
0 & (1 - 0.5z^{-1})(1 + 0.5z^{-1})
\end{bmatrix}
\]

Multi-loop minimum variance controller (calculated for each loop separately)

Two choices of GMV weightings:

(1) MV weightings

\[
P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(2) Static GMV weightings

\[
P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_c = -D^{-1} \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}
\]

interactor
Benchmarking Results

Output and control variances for different values of $K_{12}$:

<table>
<thead>
<tr>
<th>$K_{12}$</th>
<th>Controller</th>
<th>$\text{Var}[y_1]$</th>
<th>$\text{Var}[y_2]$</th>
<th>$\text{Var}[u_1]$</th>
<th>$\text{Var}[u_2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MV</td>
<td>1.3871</td>
<td>1.5514</td>
<td>0.7989</td>
<td>0.3020</td>
</tr>
<tr>
<td></td>
<td>GMV</td>
<td>2.8245</td>
<td>2.1137</td>
<td>0.1822</td>
<td>0.0144</td>
</tr>
<tr>
<td>5</td>
<td>MV</td>
<td>1.3895</td>
<td>1.5514</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>GMV</td>
<td>3.2303</td>
<td>1.9146</td>
<td>0.2078</td>
<td>0.0049</td>
</tr>
<tr>
<td>10</td>
<td>MV</td>
<td>1.4038</td>
<td>1.5514</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>GMV</td>
<td>3.3870</td>
<td>1.8782</td>
<td>0.2409</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

MV and GMV controller performance indices

- GMV criterion provides a means of balancing error and control variances
- It also makes the benchmark realizable (the plant is non-minimum phase for $K_{12} > 2$)
- MV benchmark close to 1 for small interactions
- the existing controller not so good if control variances considered
Restricted-Structure Controllers

Existing controller: filtered multi-loop PID tuned using Ziegler-Nichols rules separately for each loop.

The following RS-GMV controllers have been designed using the GMV weightings as in the previous slides:

- RS full:
  \[
  \begin{bmatrix}
  PID & PID \\
  PID & PID \\
  \end{bmatrix}
  \]

- RS diagonal #1:
  \[
  \begin{bmatrix}
  PID \\
  PID \\
  \end{bmatrix}
  \]

- RS diagonal #2:
  \[
  \begin{bmatrix}
  PID \\
  PID \\
  PID \\
  \end{bmatrix}
  \]

- RS upper triangular:
  \[
  \begin{bmatrix}
  PID & PID \\
  PID \\
  \end{bmatrix}
  \]

- RS lower triangular:
  \[
  \begin{bmatrix}
  PID & PID \\
  PID & PID \\
  \end{bmatrix}
  \]
RS Benchmarking Results

The existing multi-loop PID has been assessed against the RS controllers:

<table>
<thead>
<tr>
<th>Controller</th>
<th>$J_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-loop PID</td>
<td>2.8026</td>
</tr>
<tr>
<td>RS full</td>
<td>0.0154</td>
</tr>
<tr>
<td>RS diagonal #1</td>
<td>0.0859</td>
</tr>
<tr>
<td>RS diagonal #2</td>
<td>0.5022</td>
</tr>
<tr>
<td>RS upper triangular</td>
<td>0.0357</td>
</tr>
<tr>
<td>RS lower triangular</td>
<td>0.0658</td>
</tr>
</tbody>
</table>

- The results show the potential for improving the current performance
- I/O pairing: OK
- If further reduction in variance required, additional feedback between output 1 and input 2 (rather than between output 2 and input 1) would bring greater improvement
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Summary

- Benchmarking multivariable loops
- Benchmarking multi-loop controllers
- Detecting underperforming loops
- Tuning guidelines for the existing controllers
- Determining the optimal controller structure (I/O pairing)